Constructive methods for elliptic equations, by R. P. Gilbert, Springer Lecture Notes, no. 365, 1974, vii+397 pp.

In the last decades there has been a veritable explosion of research in partial differential equations. Much of the work involves applications of techniques and results of functional analysis, both linear functional analysis and topological results such as fixed point theorems. Somewhat apart from this main stream of investigations, there has been vigorous development in a direction which may be thought of as originating with Beltrami's representation of symmetric potentials in the last century and E. T. Whittaker's representation of solutions of Laplace's equation in three dimensions early in the present century. Bergman's integral operators, Vekua's generalised analytic functions, the present author's early work on singularities of harmonic functions are typical of this direction. Some of these developments were described in Gilbert's earlier book *Function theoretic methods in partial differential equations*, Academic Press, 1969.

The present work continues the account. It is based on lectures given by the author at the University of Delaware. Primarily these lectures were intended as a report on recent research by the group led by Gilbert at Indiana University and accordingly, they include much material not hitherto available in book form, and some material not at the time available in any printed form. Some discussion of the relation of this research to other investigations was also included, thus increasing the usefulness of these lecture notes.

A linear differential operator of the second order in two independent variables whose principal part is the Laplacian can be reduced to the form

$$L = \frac{\partial^2}{\partial z\, \partial \xi} + D(z, \xi)\frac{\partial}{\partial \xi} + F(z, \xi),$$

where $z = x + iy$ and $\xi = x - iy$ are regarded as independent complex variables. Solutions of $L[V] = 0$ can be represented in the form

$$V(z, \xi) = \int_{-1}^{1} E(z, \xi, t) f(\tfrac{1}{2}z(1-t^2))(1-t^2)^{-1/2}\, dt.$$

There are other, related, forms involving the Riemann function associated with the (formally) hyperbolic operator L. A thorough study of E leads to an analysis of the correspondence between analytic functions f of a single complex variable and solutions of $L[V] = 0$. In particular cases this approach can be utilized in boundary value problems involving the equations concerned, especially for the construction of approximate solutions (including numerical approximations). The method can be extended to elliptic equations of higher order and to a somewhat lesser extent to equations in more than two independent variables. It can be used for an analysis of the singularities of the solutions.

Function-theoretic methods can also be used for semilinear equations. This has been shown for certain second-order problems in the author's earlier book and is now extended to certain systems of equations of higher orders.