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BOOK REVIEWS

Stability of solutions of differential equations in Banach space, by Ju. L. Daleckiı and M. G. Kreĭn, Translations of Mathematical Monographs, Volume 43, American Mathematical Society, Providence, Rhode Island, 1974, vi + 386 pp. $36.40.

If challenged to describe the subject-matter of this book while standing on one foot, one might say: It is the study of the behavior (mostly at infinity) of the solutions of equations of the form $dx/dt=F(t, x)$ living in a Banach space (often a Hilbert space), with F linear in x or nearly so, and of almost all imaginable cognate matters. One might quickly state a few things the book does not do: it does not treat unbounded operators or strongly nonlinear ones; it is not interested in results specific to non-Hilbert phase space; there is little or nothing on solutions belonging to various function spaces; there are no functional-differential equations. If at that point one were to set down the other foot, muttering: But everything else is there, one might be confident of having met the challenge.

The authors have patiently accumulated over many years all the information on their topic that they could lay their hands on or, in many cases, generate themselves. The book grew through several stages of papers and lecture notes, unpublished and published, from 1947 to 1964, had a period of incubation, and emerged in 1970 in its present intricately integrated shape; some finishing touches appear only in this 1974 translation.

Stability theory is of course an almost classical branch of analysis at present and has been very active since Poincaré and Ljapunov. The specific point of view that informs the bulk of this book evolved out of the work of Ljapunov, to whom we owe our main concept of stability and who developed many of the tools still in use. It was, in my view, decisively influenced by two further events: Perron's discussion **[11]** (1930) of the relationship between the existence properties of the inhomogeneous linear equation and the exponential asymptotic behavior of the solutions of the homogeneous equation, and M. G. Kreĭn's observation (1947)—made independently by Bellman (1948)—that methods of functional analysis were available—at first to prune away unsightly and artificial computations, and then to deepen the understanding of the problems and to allow a more powerful attack on them. The possibility of dealing with equations in infinite-dimensional spaces was but a by-product of this insight.

To these two turning-points the authors would feel compelled to add a third: the long-neglected memoir of Bohl **[2]**. The authors record their amazement at the fact that this work, though published in the Crelle Journal in 1913, was for so long overlooked and that so many of its incisive contributions were rediscovered, some quite often (note, e.g., the scornful