

APPLICATIONS OF BANACH IDEALS OF OPERATORS

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For Stacy and Dana

1. **Apologies.** Since much of the recent work in the Banach space aspects of Functional Analysis, especially the geometry of Banach spaces, could, by a bit of chicanery, be construed as applications of Banach ideals, the title does not indicate a complete survey. This work is surely not exhaustive of the subject matter. Thus, many good papers are totally ignored. This is somewhat compensated for by the monograph [1.1] of Lindenstrauss and Tzafriri on the geometry of the classical Banach spaces and the “pre-book” [1.2] of A. Pietsch on the general theory of ideals of operators.

Since a lecture should have a central theme, I have chosen a fundamental result of Grothendieck which asserts that there are Banach spaces E and F for which every bounded linear operator from E to F is 2-absolutely summing. (Definitions will be forthcoming.) This result and the local structure of Banach spaces are the unifying topics of this paper. For the numerous topics this unification omits, again, apologies.

I have addressed myself to the material at hand twice before [1.3], [1.4], [1.5], the latter in collaboration with Y. Gordon and D. R. Lewis. I apologize for mentioning, again, the beautiful result of Stegall and Lewis [1.6] and tramping once again over ground covered in [1.3]–[1.5]. However, I feel, perhaps with prejudice, that these results are worthy of further discussion.

An additional apology of sorts is needed. I have included many definitions which are old hat to experts in Banach space theory. It is hoped that the material will thus be accessible to a larger audience, perhaps even to some persons completely outside Functional Analysis.

Finally, many of the results stated have their natural setting in Probability Theory. I have avoided the probabilistic language entirely. Thus “random variable” becomes “measurable function” etc. This is an editorial judgment on my part, and apologies are extended to anyone this may offend.

This is an expanded version of an invited address given at the meeting of the Society in Nashville, Tennessee, November 8, 1974.

AMS (MOS) subject classifications (1970). Primary 46–02, 47–02, 46C05, 46B10, 46B15, 46E05, 46E15, 46E30, 47B10.

Key words and phrases. Banach ideals of operators, type, cotype, stability, LUST, p -absolutely summing, nuclear, integral, \mathfrak{S}_p -, \mathfrak{D}_p - and \mathcal{G}_p -spaces.

¹This paper was prepared under grant GP-34193 of the National Science Foundation (U.S.A.).

²Portions of this paper were written during a visit to the Institut für Angewandte Mathematik der Universität Bonn. The author gratefully acknowledges partial support from the Sonderforschungsbereich 72.