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HOMOTOPY TREES FOR PERIODIC GROUPS

BY M. N. DYER

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Let π be a finite periodic group of order n whose cohomology has minimal period k. We say that π has *free period* h if π admits a periodic free resolution of the trivial π-module $\mathbf{Z}$ of length h:

$$0 \longrightarrow \mathbf{Z} \longrightarrow \mathbf{Z}\pi \longrightarrow C_{h-2} \longrightarrow C_{h-3} \longrightarrow \cdots \longrightarrow C_1 \longrightarrow \mathbf{Z}\pi \xrightarrow{\epsilon} \mathbf{Z} \longrightarrow 0$$

where each C_i is a finitely-generated free π-module. Acccording to [7], every finite periodic group of minimal period k has a minimal free period $h = pk$ for some integer $p > 0$. A convenient listing of all finite periodic groups is given in [9].

DEFINITION. A (π, m)*-complex* is a finite, connected m-dimensional CW complex X with fundamental group π whose universal cover $\widetilde{X}$ is $(m-1)$-connected.

Let $HT(\pi, m)$ denote the set of homotopy types of (π, m)-complexes. This set may be described as a *directed tree* with one vertex for each homotopy class $[X]$ of (π, m)-complexes having the homotopy type of X; the vertex $[X]$ is connected by an edge to vertex $[Y]$ provided Y has the homotopy type of the sum $X \vee S^m$ of X with the m-sphere S^m. $HT(\pi, m)$ is *connected* by [11, Theorem 14] and clearly contains no circuits.

The purpose of this note is to announce a complete description of the homotopy tree $HT(\pi, m)$ for certain periodic π and for $m = ik, ik-1$ $(i > 0)$. Full details and a description for any periodic π will appear elsewhere.

Before stating the theorem, we need two more pieces of notation. Let $\mathbf{Z}_n^*$ be the units of the ring $\mathbf{Z}_n$ of integers modulo n. Then $\mathrm{Aut}_k\,\pi = \{p \in \mathbf{Z}_n^* \mid \exists \alpha \in \mathrm{Aut}\,\pi \ni \alpha_k^*(1) = p$ where $\alpha_k^*\colon H^k(\pi, \mathbf{Z}) \longrightarrow H^k(\pi, \mathbf{Z})\}$. Let $\widetilde{K}_0\mathbf{Z}\pi$ be the reduced projective class group of the integral group ring $\mathbf{Z}\pi$ of π. Define a homomorphism $\nu\colon \mathbf{Z}_n^* \longrightarrow \widetilde{K}_0\mathbf{Z}\pi$ by $\nu(p) =$ class of the projective left ideal (p', N) of $\mathbf{Z}\pi$ generated by any integer $p' \in p$ and $N = \Sigma_{x \in \pi} x$. ν is well defined by [7, Lemma 6.1].

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