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ACOUSTIC BOUNDARY CONDITIONS

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In this paper we announce results in the study of the wave equation

$$\phi_{tt} = \Delta\phi \tag{1}$$

subject to what we call acoustic boundary conditions. The physical model giving rise to these conditions is that of a gas undergoing small irrotational perturbations from rest in a domain G with smooth compact boundary. We assume that each point of the surface ∂G acts like a spring in response to the excess pressure in the gas, and that there is no transverse tension between neighboring points of ∂G, i.e., the "springs" are independent of each other. (Such a surface is called locally reacting; see [2, pp. 259–264].)

If the boundary has mass per unit area m, resistivity d, and spring constant k (all nonnegative functions on ∂G), then the displacement $\delta(x, t)$ into the domain of a point $x \in \partial G$ at time t must satisfy the spring equation

$$m\delta_{tt} + d\delta_t + k\delta = - \text{excess pressure} = \rho_0\phi_t, \tag{2}$$

where ρ_0 is the unperturbed density of the gas and $\phi(x, t)$ is the velocity potential. Continuity of the normal velocity between the gas and the boundary implies the relation $\delta_t(x, t) = -\phi_n(t, x - n\delta(x, t))$, $x \in \partial G$, where n is the outward normal. We consider here the linearized approximation obtained by assuming δ is small (this is consistent with the linearization leading to the wave equation). Thus we assume

$$\delta_t(x, t) = -\phi_n(x, t). \tag{3}$$

Note that if d and k are zero, (2) and (3) imply $m\phi_{nt} + \rho_0\phi_t = 0$; thus the excess pressure satisfies the Robin boundary condition.

If ϕ and δ are smooth solutions of (1)–(3), and ϕ has compact support in space for each t if G is unbounded, it is easy to see that the energy form

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