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SPECTRAL THEORY FOR BOUNDARY VALUE PROBLEMS FOR ELLIPTIC SYSTEMS OF MIXED ORDER

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Introduction. For a closed, densely defined linear operator T in a Hilbert space H, we define the essential spectrum ess sp T as the complement in $\mathbf{C}$ of the set of λ for which $T - \lambda$ is a Fredholm operator (with possibly nonzero index). Recall (cf. Wolf [7]) that $\lambda \in$ ess sp T if and only if either $T - \lambda$ or $T^* - \bar{\lambda}$ has a singular sequence, i.e. a sequence $u_k \in H$ with $\|u_k\| = 1$ for all k, $(T - \lambda)u_k \longrightarrow 0$ (or $(T^* - \bar{\lambda})u_k \longrightarrow 0$) in H, but u_k having no convergent subsequence in H. ess sp T is closed and invariant under compact perturbations of T, and contains the accumulation points of the eigenvalue spectrum.

Let $\bar{\Omega}$ be an n-dimensional compact C^∞ manifold with boundary Γ and interior $\Omega = \bar{\Omega}\backslash\Gamma$. It is well known that when A is a properly elliptic operator on $\bar{\Omega}$ of order $r > 0$, the L^2-realization $A_B : u \longmapsto Au$ with domain $D(A_B) = \{u \in L^2(\Omega) | Au \in L^2(\Omega), Bu|_\Gamma = 0\}$, defined by a boundary operator B that covers A (i.e. $\{A, B\}$ defines an elliptic boundary value problem), has ess sp $A_B = \emptyset$.

However, when A is a *system* of *mixed order*, elliptic in the sense of Douglis and Nirenberg (cf. [1]), ess sp A_B can be nonempty even when $\{A, B\}$ is elliptic with smooth coefficients and $\bar{\Omega}$ is compact. We study this phenomenon for a class of Douglis-Nirenberg systems of nonnegative order, determine the essential spectrum, and find the asymptotic behavior of the discrete spectrum at $+\infty$ for the selfadjoint lower bounded realizations.

Examples of the systems we consider are: The linearized Navier-Stokes operator and certain systems stemming from nuclear reactor analysis. A preliminary, less advanced account of the theory was given in [5].

1. Preliminaries.

1.1. For q integer > 1 there is given a set of integers $m_1 \geqslant m_2 \geqslant$

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