

EVERY CLOSED ORIENTABLE 3-MANIFOLD IS A 3-FOLD BRANCHED COVERING SPACE OF S^3

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Let $p: M \rightarrow N$ be a nondegenerate simplicial map between compact triangulated manifolds of the same dimension n . This is a branched covering space if the restriction of p , called \tilde{p} , gives a covering space map $\tilde{p}: M - (n - 2 \text{ skeleton}) \rightarrow N - (n - 2 \text{ skeleton})$. The set of points x in M such that p does not map any neighborhood of x homeomorphically into N is called the branch cover B . The $(n - 2)$ -dimensional set $p(B)$ is called the branch set. J. W. Alexander asserted the following theorem [1].

ALEXANDER'S THEOREM. *Every closed orientable 3-manifold is a branched covering space of S^3 with the branch set a link in S^3 .*

For a proof see [5].

The purpose of this paper is to announce the following result.

THEOREM 1. *Every closed orientable 3-manifold is an irregular 3-sheeted branched covering of S^3 with branch set a knot.*

We shall only sketch the proof of the theorem. A detailed proof will appear elsewhere.

The main part of the proof consists of constructing a certain irregular 3-fold branched covering of the 3-ball, D^3 , by a handlebody of genus g , X_g , with branch set A , a set of $g + 2$ proper arcs, and observing that for this particular branched covering any homeomorphism ψ of ∂X_g is isotopic to a homeomorphism ϕ that projects to a homeomorphism ϕ of ∂D^3 . ϕ necessarily leaves the branch set $A \cap \partial D^3$ invariant as a set. Moreover we can choose ψ and ϕ so that ψ induces a cyclic permutation on the branch set. Let $i(i)$ be the map that identifies X_g with X'_g (D^3 with $D^{3'}$) restricted to the

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