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ISOMETRIC MINIMAL IMMERSIONS OF $S^3(a)$ IN $S^N(1)$

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Introduction. We denote by $S^p(a)$ the sphere of radius **a** in the euclidean $(p + 1)$-space E^{p+1}, with the induced metric. In [**1**], S. S. Chern asks the following question: "Let $S^3(a) \to S^7(1)$ be an isometric minimal immersion. Is it totally geodesic?". In this note we announce the following result.

THEOREM. *Let $S^3(a) \subset E^4 \to S^N(1) \subset E^{N+1}$ be an isometric minimal immersion which is not totally geodesic. Then $N \geqslant 8$.*

The class of isometric minimal immersions of $S^p(a) \to S^N(1)$ was qualitatively described by M. do Carmo and N. R. Wallach in [**3**]. For $p = 2$, each admissible **a** determines a unique element of such a class. The main result of [**3**] shows that for each $p \geqslant 3$ and each admissible $a \geqslant \sqrt{8}$, there exists a continuum of distinct such immersions. Our Theorem is an answer to a question of quantitative character. This constitutes part of our doctoral dissertation at IMPA. I want to thank my adviser Professor M. do Carmo for suggesting this problem and for helpful conversations.

Definitions and lemmas. Let $H = (\varphi_0, \cdots, \varphi_N)$: $S^3(a) \subset E^4 \to S^N(1) \subset E^{N+1}$ be an isometric minimal immersion. Then [**1**] the coordinate functions are spherical harmonics on $S^3(a)$, i.e., each φ_i $(0 \leqslant i \leqslant N)$ is the restriction to $S^3(a)$ of a homogeneous polynomial of degree s, with four indeterminates satisfying the condition

$$\sum_{k=1}^{4} \frac{\partial^2 \varphi_i}{\partial x_k^2} = 0. \tag{1}$$

Initially we set

$$\varphi_i = \sum_{\Sigma \alpha_i = s} a_{\alpha_1 \cdots \alpha_4} x_1^{\alpha_1} \cdots x_4^{\alpha_4}, \tag{2}$$

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