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H-SPACES WITH FINITELY GENERATED COHOMOLOGY ALGEBRAS

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Introduction. The purpose of this note is to announce several results which describe the mod p cohomology ring of an H-space. We assume for the rest of the paper that we are dealing with connected H-spaces with the homotopy type of a CW complex having finitely many cells in each dimension. We use the notation Z_p to denote Z/pZ. Then the cohomology $H^*(X; Z_p)$ is a graded, connected Hopf algebra of finite type, and the cohomology and homology mod p are dual Hopf algebras. If A is a Hopf algebra, $P(A)$ and $Q(A)$ will denote the module of primitives and indecomposables respectively.

There is a secondary cohomology operation which detects the dual of a homology pth power in the mod p cohomology of an H-space. Often, the secondary operation will show that either there is an infinite sequence of nonzero even-dimensional generators of increasing dimension in the cohomology ring, or a given generator is in the image of primary operations occurring in the indeterminacy.

For finite H-spaces, the secondary operation can be used to prove that the third homotopy group has no odd torsion and has two torsion of order at most two. For H-spaces having finitely generated cohomology and no p-torsion of order p, the secondary operation shows that the even generators are concentrated in dimension two for p an odd prime.

A theorem of Milnor and Moore [6] implies that the mod p cohomology of an H-space $X, H^*(X; Z_p)$, is primitively generated if and only if the homology $H_*(X; Z_p)$ is commutative and associative and every element has height less than or equal to p. For H-spaces having $(QH)^{\text{even}}(X; Z_p)$ finite dimensional and $\beta_1(QH)^{\text{even}}(X; Z_p) = 0$, we show that $H^*(X; Z_p)$ is primitively generated if and only if $H_*(X; Z_p)$ is commutative and associative for p odd.

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