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GAUSS FORMULAS FOR THE DIRICHLET PROBLEM

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The 2-dimensional Dirichlet problem for Laplace's equation is the problem of finding $u(x, y)$ to satisfy

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad \text{in } R, \qquad u(x, y) \text{ given on } B. \tag{1}$$

Here R is an open, bounded, simply-connected region in the (x, y)-plane and B is the boundary of R. We assume that B is a rectifiable Jordan curve.

It is well known (see, for example, Protter and Weinberger [4, p. 85]) that the solution of (1) at a point (x_*, y_*) in R can be written

$$u(x_*, y_*) = \int_B w(x, y; x_*, y_*) u(x, y)\, ds, \tag{2}$$

where

$$w(x, y; x_*, y_*) \equiv -\frac{\partial G}{\partial n}(x, y; x_*, y_*)$$

is the normal derivative of Green's function for R.

We are interested in approximations of the form

$$u(x_*, y_*) \simeq \sum_{k=1}^{N} A_k u(x_k, y_k) \tag{3}$$

where the A_k are real constants and the (x_k, y_k) are points on B. We say that (3) is a *harmonic interpolation formula of degree* d if (3) is an equality for all harmonic polynomials of degree $\leqslant d$, and if there is at least one harmonic polynomial of degree $d + 1$ for which (3) is not an equality. A harmonic polynomial of degree d is any linear combination of the linearly independent polynomials

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