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THE PRESSURE IS INDEPENDENT OF THE BOUNDARY CONDITIONS FOR $P(\phi)_2$ FIELD THEORIES

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This research announcement represents a continuation of our program [6] of applying statistical mechanical methods to study the $P(\phi)_2$ Euclidean quantum field theory [11], [18], [15]. Our main results are:

(a) The pressures and the ground state energies for different boundary conditions converge to the same infinite volume limit.

(b) The Gibbs variational equality for the entropy density is satisfied.

(c) For interactions of the type $P(x) = \lambda x^4 - \mu x$ with $\lambda > 0$ and $\mu \neq 0$, the infinite volume Dirichlet theory has a mass gap.

The free field of mass $m > 0$ (in two dimensions) is the Gaussian process ϕ indexed by $S(\mathbf{R}^2)$ with variance

$$\int \phi(f)^2 d\mu_0 = \langle f, (-\Delta + m^2)^{-1} f \rangle, \tag{1}$$

where $\langle \cdot , \cdot \rangle$ denotes the $L^2(\mathbf{R}^2)$ inner product and Δ is a two-dimensional Laplacian. It is convenient to realize $d\mu_0$ as a Borel measure on $S'(\mathbf{R}^2)$ [15]. Given a bounded rectangle Λ in $\mathbf{R}^2$, we introduce three additional Gaussian processes, indexed by $C_0^\infty(\Lambda)$, with variance given by the analogue of (1) but with Δ replaced by the Laplacian on $L^2(\Lambda)$ with Dirichlet, Neumann or periodic boundary conditions. We denote the corresponding measures by $d\mu_{0,\Lambda}^X$, $X = D, N$, or P. (In the cases of Dirichlet and Neumann boundary conditions, Λ may be taken to be any bounded open region.)

Let P be a polynomial bounded below as a function on $\mathbf{R}$ and normalized by $P(0) = 0$. For $\Lambda \subset \mathbf{R}^2$ a bounded rectangle, define the inter-

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