BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 80, Number 6, November, 1974

A CONJECTURE OF M. GOLOMB ON OPTIMAL AND NEARLY-OPTIMAL LINEAR APPROXIMATION

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Communicated by M. Golomb, May 28, 1974

In 1964, M. Golomb, in his survey paper on optimal and nearly-optimal linear approximation, presented at the General Motors Conference [3], called attention to an unsolved problem. It is the purpose of this note to solve this problem and at the same time to give a certain extension of the Haršiladze-Lozinskiĭ theorem.

The authors are indebted to Professor P. L. Butzer for many helpful discussions and for a critical reading of the manuscript.

Let $C_{2\pi}$ be the space of continuous 2π-periodic functions with Čebyšev norm, Π_n the class of trigonometric polynomials of degree $\leqslant n$, and $E_n[f] = \inf\{\|f-p\|; p \in \Pi_n\}$ the error of best approximation of an $f \in C_{2\pi}$ by elements of Π_n for an $n \in \mathbf{P} = \{0, 1, 2, \cdots\}$. A sequence $\{U_n\}_{n \in \mathbf{P}}$ of bounded linear operators on $C_{2\pi}$ into $C_{2\pi}$ is called *asymptotically optimal* [3] for a given subset $Y \subset C_{2\pi}$ if

$$(1) \qquad \sup_{f \in Y} \|f - U_n f\| \leqslant M_Y \sup_{f \in Y} E_n[f] \qquad (n \in \mathbf{P}),$$

M_Y being some constant. $\{U_n\}$ is called *optimal* for Y if (1) is satisfied with $M_Y = 1$.

In particular, Y will be taken to be one of the spaces $C_0^r, r \in \mathbf{P}$ or $A_0^\alpha, \alpha > 0$, where C_0^r consists of those $f \in C_{2\pi}$ whose rth derivative is continuous and satisfies $\|f^{(r)}\| \leqslant 1$, and A_0^α is the class of functions $f(z)$ of a complex variable $z = x + iy$ which are 2π-periodic in x, real for $y = 0$, analytic in the open strip $|y| < \alpha$, continuous in $|y| \leqslant \alpha$, and satisfy

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AMS (MOS) subject classifications (1970). Primary 42A08; Secondary 41A25, 41A35, 41A50.

Key words and phrases. Best trigonometric approximation, linear polynomial operators, optimal rate of approximation, Haršiladze-Lozinskiĭ theorem.

[1] Supported by a DFG research grant (Bu 166/21) which is gratefully acknowledged.