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MANIFOLDS WITH FUNDAMENTAL GROUP A GENERALIZED FREE PRODUCT. I

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This announcement describes new methods in the study and classification of differentiable, Pl or topological manifolds with infinite fundamental group. If, for example, Y^{n+1} is a closed manifold with $\pi_1 Y = G_1 *_H G_2$, $n \geqslant 4$, there is a decomposition $Y = Y_1 \cup_X Y_2$ with $\pi_1(Y_1) = G_1, \pi_1(Y_1) = G_2, \pi_1(X) = H$. For large classes of fundamental groups, the codimension one splitting theorems of [C3], extending results of [B1], [B2], [BL], [FH], [L], [W2], reduced the classification of manifolds homotopy equivalent to Y to the classifications of the manifolds homotopy equivalent to Y_1 and to Y_2. However, the construction in [C4] and [C5] of manifolds V simple and tangentially homotopy equivalent to $RP^{4k+1} \# RP^{4k+1}$, $k > 0$, but V is *not* a nontrivial connected sum, demonstrated in a strong way the existence of unsplittable manifolds and homotopy equivalences.[2]

In the present announcement we get around these difficulties by, roughly speaking, adapting methods of [C3] to construct an abelian group we call the *unitary nilpotent group* which depends on H, G_1, G_2 and which acts *freely* on the set of manifolds equipped with homotopy equivalences to Y, with each coset of this action containing a *unique* split manifold. Thus, the classification of manifolds homotopy equivalent to Y is reduced to computing UNil groups and to classifying split homotopy equivalences. In this setting, all earlier splitting theorems are reinterpreted as showing that for certain H, G_1, G_2, the unitary nilpotent group vanishes. However, for $H = 0$, $Z_2 \subset G_1$, $G_2 \neq 0$, $n = 4k$ and Y orientable, the corresponding unitary nilpotent group is not finitely generated [C4], [C6].

The unitary nilpotent groups are *2-primary* and are defined algebraically in [C6] and depend only on the ring with involution $Z[H]$, the $Z[H]$-bi-

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[2] Note also our construction in [C4] and [C5] of counterexamples to a splitting theorem of Miščenko [M].