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NONSELFADJOINT FOURTH ORDER DIFFERENTIAL EQUATIONS WITH CONJUGATE POINTS

BY KURT KREITH

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Much of the classical Sturm oscillation theory extends to selfadjoint differential equations of order $2n$ if the notion of "oscillation" is formulated in terms of conjugate points determined by nontrivial solutions with nth order zeros at two distinct points. The standard techniques for achieving such generalizations also allow one to establish *dis*conjugacy criteria in the nonselfadjoint case (see [1]–[4]), but I know of no corresponding criteria for the existence of conjugate points when the equation under consideration is not selfadjoint.

The purpose of this note is to sketch a technique which deals with the sufficiently regular general real fourth order equation

$$l[y] \equiv (p_2(t)y'' - q_2(t)y')'' - (p_1(t)y' - q_1(t)y)' + p_0(t)y = 0, \tag{1}$$

and establishes the existence of a $\beta > \alpha$ such that

$$y(\alpha) = y'(\alpha) = 0 = y(\beta) = y'(\beta)$$

is satisfied by a nontrivial solution $y(t)$ of (1). A detailed proof will appear elsewhere.

We begin with an oscillation preserving transformation used by the author [5] to eliminate the third order term $(q_2(t)y')''$. Generalizing upon a technique used by Whyburn [6] for selfadjoint equations, one can then obtain a representation of (1) in the form

$$y'' = a(t)y + b(t)x, \qquad x'' = c(t)y + d(t)x \tag{2}$$

where $b(t) = 1/p_2(t) > 0$, and the formulas for the other coefficients of (2) are given in [5]. The system (2) allows an obvious dynamical interpretation in terms of a particle of unit mass moving in the x, y-plane. By an

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