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THE HOPF RING FOR COMPLEX COBORDISM[1]

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It is our purpose here to announce the results of our study of the homology of the spaces in the Ω-spectrum for complex cobordism and Brown-Peterson cohomology. Let $MU(n)$ be the standard Thom complex. $MU_k = \lim_{n\to\infty} \Omega^{2(n-k)}MU(n)$ is the $2k$ space in the Ω-spectrum for complex cobordism. We will consider the space $\mathbf{M}U = \lim_{n\to-\infty} \Pi_{j>n} MU_j$. We find this product easier to study than the separate factors, as will become apparent below.

For a space X we have $[X, \mathbf{M}U] \simeq U^{2*}(X)$, the even degree part of the complex cobordism of X. Because MU is a multiplicative theory, $U^{2*}(X)$ is a ring and $\mathbf{M}U$ is a commutative ring with identity in the homotopy category. Thus we have that for any field k, $H_*(\mathbf{M}U; k)$ is a commutative ring with identity in the category of k-coalgebras, i.e., it is a "Hopf ring".

In more common language, the homology has two products and a coproduct. $\circ$ will denote the multiplicative product which comes from the ring structure on the spectrum, while $*$ will denote the additive product coming from the loop structure $(\Omega^2\mathbf{M}U \simeq \mathbf{M}U)$. They obey the following distributive law: if $\psi(z) = \Sigma\, z' \otimes z''$ is the coproduct, then $z \circ (x * y) = \Sigma\, (z' \circ x) * (z'' \circ y)$.

We now describe the structure of $H_*(\mathbf{M}U; R)$ where R is an algebra over a field k. Let

$$C_R(X) = \left\{x \in \prod_{i\geq 0} H_i(X; R) \colon \psi(x) = x \mathbin{\hat{\otimes}} x, x \neq 0\right\}.$$

$C_R(\mathbf{M}U)$ is a ring, and for each $x \in C_R(X)$ we have a ring homomorphism $\lambda_x\colon U^{2*}(X) \longrightarrow C_R(\mathbf{M}U)$ defined by $\lambda_x(u) = u_*(x)$ for $u \in U^{2*}(X)$. Let

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