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A CRITERION FOR THE EXISTENCE OF BIHARMONIC GREEN'S FUNCTIONS

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The harmonic Green's function was originally introduced as the electrostatic potential of a point charge in a grounded system. Its characterization by the fundamental singularity and vanishing boundary values permitted its generalization to regular subregions Ω of an abstract Riemann surface or Riemannian manifold R. The Green's function g on R was then defined as the directed limit, if it exists, of the Green's function g_Ω on Ω as $\{\Omega\}$ exhausts R. The distinction of Riemann surfaces and Riemannian manifolds into hyperbolic and parabolic types according as g does or does not exist is still a cornerstone of the harmonic classification theory.

The biharmonic Green's function γ also has an important physical meaning: it is the deflection of a thin elastic plate under a point load. However, in sharp contrast with the harmonic case, nothing seems to be known about its existence on noncompact spaces. The purpose of the present paper is to initiate research on this fundamental problem of biharmonic classification theory.

Biharmonicity being not meaningful on abstract Riemann surfaces, our aim is to generalize the definition of the biharmonic Green's function to Riemannian manifolds R and to explore its existence on them. On a regular subregion Ω of R, there exist two biharmonic Green's functions, denoted by β and γ, with a biharmonic fundamental singularity, and with boundary data $\beta = \partial\beta/\partial n = 0$ and $\gamma = \Delta\gamma = 0$. For dimension 2, both functions give the deflection under a point load of a thin plate which is clamped or simply supported at the edges, respectively. Our present investigation deals exclusively with γ. The corresponding function γ_Ω on Ω increases with Ω, and we set $\gamma = \lim_{\Omega \to R} \gamma_\Omega$ on R.

We first study the existence of γ on the Euclidean N-space R^N. The result turns out to be quite fascinating: γ exists if and only if $N > 4$. By

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