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CONDITIONAL INDEPENDENCE

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1. **Introduction.** One of the basic concepts which pervades the whole of probability theory is that of independence. This is the condition which serves as part of the hypothesis of most of the major theorems of probability, most notably the laws of large numbers. In many cases, this condition is weakened to *stationarity* or *uncorrelatedness*, and usually there is a corresponding weakening in the sharpness of results. We shall define some new conditions which are weakenings of independence, and which yield many of the usual theorems in real or complex random variables, but which have noticeably weaker effect when applied to Banach space-valued random variables.

The study of probability in Banach spaces enables us to distinguish between the effects of probabilistic assumptions and those which flow from the geometry of the spaces in which the values are taken. In the strong law of large numbers, the extremes are the strong law of E. Mourier (1953) which holds in all Banach spaces, and the Kolmogorov strong law, which holds in Hilbert spaces and some others (these theorems are quoted below).

The new conditions defined here are properly weaker than independence, though some of them are sufficient to give the Kolmogorov strong law of large numbers in Hilbert spaces. The difference for Banach space-valued random variables, however, is striking.

2. **Basic notions.** We define random variables in Banach spaces by a natural extension of their definitions in the real case (cf., e.g., Beck [1], [2], [3]). In the same way, we define expectation, independence, stationarity, distribution, and symmetric random variable, in a natural way, and define, for each random variable with an expectation, the variance as $\mathrm{Var}(X) = \sigma^2(X) = E(\|X - E(X)\|^2)$. As a generalization of *orthogonality* or *uncorrelatedness*, we define two random $\mathfrak{X}$-variables as *weakly orthogonal* if

$$E(x^*(X - E(X))x^*(Y - E(Y))) = 0, \qquad \forall x^* \in \mathfrak{X}^*.$$

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