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ACCESSIBILITY AND FOLIATIONS WITH SINGULARITIES

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Introduction. Recently, Sussmann has proved that the accessible sets of a system of vectorfields on a C^∞ manifold M are immersed submanifolds of M [**1**], [**2**]. In this paper we state a general theorem on accessible sets of collections of 'arrows' and indicate how it implies (a) the above result; (b) the fact that the orbits of an arbitrary 'isotopically connected' subgroup of Diff(M) form a foliation with singularities; and (c) a similar result for groupoids of germs of local diffeomorphisms. The complete proofs will be published elsewhere.

The results of this paper were obtained independently of Sussmann's work.

1. **Statement of the main result.** Throughout this paper, the word 'differentiable' refers to a fixed class C^q, $1 \leqq q \leqq \infty$, and M is a finite-dimensional paracompact differentiable manifold. Theorems 1 and 4 are also valid in the real analytic case.

A subset L of M is said to be a *k-leaf* of M if there exists a differentiable structure σ on L such that (i) (L, σ) is a connected k-dimensional immersed submanifold of M, and (ii) if N is an arbitrary locally connected topological space, and $f: N \rightarrow M$ is a continuous function such that $f(N) \subset L$, then $f: N \rightarrow (L, \sigma)$ is continuous.

It follows from the properties of immersions that if $f: N \rightarrow M$ is a differentiable mapping of manifolds such that $f(N) \subset L$, then $f: N \rightarrow (L, \sigma)$ is also differentiable. In particular, σ is the unique differentiable structure on L which makes L into an immersed k-dimensional submanifold of M. Since M is paracompact, every connected immersed submanifold of M is separable, and so the dimension k of a leaf L is uniquely determined.

We say that F is a *C^q-foliation of M with singularities* if F is a partition of M into C^q-leaves of M such that, for every $x \in M$, there exists a local C^q-chart ψ of M with the following properties:

(a) The domain of ψ is of the form $U \times W$, where U is an open neighbourhood of 0 in R^k, W is an open neighbourhood of 0 in R^{n-k}, and k is the dimension of the leaf through x.

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