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WEIL REPRESENTATIONS AND CUSP FORMS ON UNITARY GROUPS

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Introduction. In this note we announce some results on the character theory of the finite unitary groups. The main result is the decomposition of the restriction of a certain class function J to a maximal parabolic subgroup as a tensor product. This result is analogous with those of [**4**] for the general linear group $\mathrm{GL}(n, q)$, with the difference that the components of the tensor product are more complicated here. In fact, one has a Levi decomposition $P=MH$ of the maximal parabolic subgroup P, where M is a unitary group of lower dimension, and H is isomorphic to the "unitary Heisenberg group" (see below); one therefore (see [**3**], [**6**], [**7**]) obtains "Weil representations" of P, and these are instrumental in the decomposition of J.

Applications include an inductive proof that the restriction of J to the unipotent subgroup of the unitary group is a proper character, and that J has a nontrivial projection to the space of cusp forms on the unitary group in odd dimensions.

In some forthcoming work of Lusztig, the main theorem is used to show that J is in fact a proper irreducible discrete series character of the unitary group, verifying a long-standing conjecture of Ennola [**1**].[1]

1. **The unitary group.** We consider the finite unitary groups realized as follows: Let G be $\mathrm{GL}(n, q^2)$, and consider the automorphism σ of G given by

$$\sigma = {}^{-} \circ -1 \circ t \circ Y,$$

where $\circ$ denotes composition, $^{-}$ is conjugation (taking the matrix elements to the qth power), t is transposing of matrices and Y is conjugation by the matrix

$$\begin{bmatrix} & & & 1 \\ & 0 & 1 & \\ & & \cdot & \\ & \cdot & & \\ \cdot & & 0 & \\ 1 & & & \end{bmatrix}.$$

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[1] *Added in proof.* The author has recently learned that Ennola's conjecture for J_n is still open.