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L^p-CONVOLUTION OPERATORS AND TENSOR PRODUCTS OF BANACH SPACES

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Associated with every locally compact group are triples of Banach algebras

$$(1) \qquad \{\mathscr{C}_0(G), L^1(G), L^\infty(G)\}, \qquad \{A(G), C^*(G), B(G)\}$$

intimately connected with duality theory (the notation is that of **[1]**). In both cases the middle algebra is the closure of $L^1(G)$ in the dual of the first algebra and also the predual of the third algebra (at least when G is amenable in the second case). Furthermore, the third algebra is closely connected with the multiplier algebra of the first algebra.

For abelian groups, compact or discrete, Varopoulos **[11]**, **[12]** showed to great effect how the second triple could be obtained and studied by starting with the tensor product $\mathscr{C}_0(G) \otimes_\gamma \mathscr{C}_0(G)$, γ the greatest cross-norm. An analogous construction starting this time with $\mathscr{C}_0(G) \otimes_\lambda \mathscr{C}_0(G)$, λ the least cross-norm, would produce the first triple. On the other hand, at least for amenable groups, the triples in (1) can be considered as the extreme case $p=1, 2$, respectively, of a family $\{A^p(G), cv^p(G), B^p(G)\}$, $1 \leqq p \leqq 2$, associated with L^p-convolution operator theory, and obtained by starting with the tensor product $L^{p'}(G) \otimes_\gamma L^p(G), p \neq 1$, or $\mathscr{C}_0(G) \otimes_\gamma L^1(G)$, $p=1$. Indeed, Herz has shown that $A^p(G)$ is a pointwise Banach algebra **[6]** while $B^p(G)$, $1<p\leqq 2$, is both the multiplier algebra of $A^p(G)$ and the Banach dual space of $cv^p(G)$, G amenable. In these notes we outline a new approach to convolution operator theory, by starting with $\mathscr{C}_0(G) \otimes_\alpha \mathscr{C}_0(G)$, α a tensorial norm **[5]**, rather than with $L^{p'}(G) \otimes_\gamma L^p(G)$. A triple $\{\mathscr{V}^\alpha(G), \mathscr{L}^{\alpha'}(G), \mathscr{W}^\alpha(G)\}$ analogous to (1) is obtained. For L^p-convolution operator theory, a family of tensorial norms α_{pq} is used. The two basic ideas are to exploit classical Banach space theory concerning $L^p(\mu)$-spaces, for example, forgetting about group structure, and then, when a group structure is imposed, to exploit standard $\mathscr{C}_0(G)$- and $L^1(G)$-techniques because all the 'L^p-theory' has been thrown into the norm α_{pq},

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