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THE COMPUTATION OF SURGERY GROUPS OF ODD TORSION GROUPS

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The purpose of this note is to describe some of the main results pertaining to the computation of surgery groups of finite groups found in a joint paper [7] with W. Scharlau, one book [1] and five papers [2]–[6] of the author. Mention of the results is made in [10]. I shall indicate the source of each result. Throughout this note π denotes a group. Let us begin with the results for surgery groups of odd torsion groups. Results closely related to the first three theorems have been announced in Wall [12].

THEOREM 1 [2]. *If π is an odd torsion group then the surgery obstruction groups*

$$L^{s,h}_{2n+1}(\pi) = 0.$$

Let r_∞ denote the number (infinite if π is infinite) of irreducible real representations of π.

THEOREM 2 [3]. *If π is an odd torsion group then the surgery obstruction groups*

$$\begin{aligned} L^s_{2n}(\pi) &= Z^{r_\infty} && \text{if } n \equiv 0 \bmod 2, \\ &= Z^{r_\infty - 1} \oplus Z_2 && \text{if } n \equiv 1 \bmod 2, \end{aligned}$$

and in the latter case the nontrivial element of Z_2 is represented by the based quadratic form $(Z\pi \oplus Z\pi, \left(\begin{smallmatrix}1 & 0\\ 1 & 1\end{smallmatrix}\right))$.

Let $Z\pi$ and $Q\pi$ be the integral and rational group rings of π. Let $K_0(Z\pi, Q\pi)$ be the relative group in the exact sequence of a localization [9, IX, §6] and let $\tilde{K}_0(Z\pi) = K_0(Z\pi)/[Z\pi]$ be the projective class group of $Z\pi$. $K_0(Z\pi, Q\pi)$ is generated by pairs (M, N) of finitely-generated projective $Z\pi$-lattices on a free $Q\pi$-module and if $M^* = \mathrm{Hom}_{Z\pi}(M, Z\pi)$, then $K_0(Z\pi, Q\pi)$ has a Z_2-action defined by $(M, N) \mapsto -(M^*, N^*)$, and $\tilde{K}_0(Z\pi)$ a Z_2-action defined by $M \mapsto -M^*$. Let $H^0(K_0(Z\pi, Q\pi))$ be the zeroth cohomology group of the Z_2-action on $K_0(Z\pi, Q\pi)$ and let

$$H(\pi) = \operatorname{coker} H^0(K_0(Z\pi, Q\pi)) \to \tilde{K}_0(Z\pi).$$

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