BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 80, Number 6, November 1974

UNBOUNDED OPERATORS WITH SPECTRAL CAPACITIES

BY I. ERDELYI

Communicated by Robert Bartle, January 15, 1974

The concept of spectral capacity introduced by C. Apostol in [1] and its relationship to decomposable operators [3] established by a theorem of C. Foiaş [4] are used for an investigation in the unbounded case.

Let $\mathfrak{S}(X)$ denote the family of subspaces (closed linear manifolds) of a Banach space X, and let $\mathfrak{F}$ and $\mathfrak{K}$ represent the collection of closed and compact subsets of the complex plane π, respectively. The superscript c stands for the complement.

1. DEFINITION [1]. A spectral capacity in X is an application $\mathfrak{E}:\mathfrak{F}\rightarrow\mathfrak{S}(X)$ which satisfies the following conditions:

(i) $\mathfrak{E}(\varnothing)=\{0\}$, $\mathfrak{E}(\pi)=X$;

(ii) $\bigcap_{n=1}^{\infty}\mathfrak{E}(F_n)=\mathfrak{E}(\bigcap_{n=1}^{\infty}F_n)$, $\{F_n\}\subset\mathfrak{F}$;

(iii) for every finite open cover $\{G_i\}_{1\leqq i\leqq m}$ of $F\in\mathfrak{F}$, $\mathfrak{E}(F)=\sum_{i=1}^{m}\mathfrak{E}(F\cap\bar{G}_i)$.

In order to confine the present investigation to densely defined operators on X, the following additional constraint on the spectral capacity is needed:

2. DEFINITION. A spectral capacity $\mathfrak{E}$ will be referred to as regular if the linear manifold

$$X_0=\{x\in\mathfrak{E}(K):K\in\mathfrak{K}\}$$

is dense in X.

3. DEFINITION. A linear operator $T:D(T)\ (\subseteq X)\rightarrow X$ is said to possess a regular spectral capacity $\mathfrak{E}$ (abbrev. $T\in\mathfrak{T}(\mathfrak{E})$) if it is closed, has a nonvoid resolvent set and satisfies the following conditions:

(iv) $\mathfrak{E}(K)\subseteq\mathfrak{D}(T)$ for all $K\in\mathfrak{K}$;

(v) $T(\mathfrak{E}(F)\cap\mathfrak{D}(T))\subseteq\mathfrak{E}(F)$ for all $F\in\mathfrak{F}$;

(vi) the restriction $T_F=T|\mathfrak{E}(F)\cap\mathfrak{D}(T)$ has the spectrum $\sigma(T_F)\subseteq F$, $F\in\mathfrak{F}$.

4. THEOREM. *Given $T\in\mathfrak{T}(\mathfrak{E})$. For every $K\in\mathfrak{K}$, the restriction $T_K=T|\mathfrak{E}(K)$ is a (bounded) decomposable operator on $\mathfrak{E}(K)$ possessing the*

AMS (MOS) subject classifications (1970). Primary 47B99; Secondary 47A15, 47B40.

Key words and phrases. Unbounded operators, spectral capacity, decomposable operators, spectral maximal spaces, weak spectral manifolds.