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A GENERAL THEORY OF IDENTITIES OF THE ROGERS-RAMANUJAN TYPE

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1. **Introduction.** This paper is a direct successor to [8], "Partition identities", henceforward referred to as PI. The lecture presented under the title of the present paper [9] can be roughly broken into three parts. Part 1 was a historical survey; Part 2 presented the basic elements of the theory of partition ideals and discussed partition ideals of order 1 since they possess rather elegant properties that are easily developed. Both Parts 1 and 2 essentially appear in §§ 1–3 of PI. Part 3 (of [9]) was devoted to a discussion of how the more difficult questions in the theory of partition identities might be attacked. In §2 we shall present a resumè of the elementary definitions and results for partition ideals. In §3 below, we shall present in detail the discussion of the Rogers-Ramanujan identities that made up [9, Part 3]. It is now possible to provide a substantial partial answer to the first of the two main questions raised in §3, and this result will be presented as Theorem 4.1 in §4. §5 will outline how the ideas involved in the proof of Theorem 4.1 have extended our knowledge of the general Rogers-Ramanujan theorem [6], [14]. In §6, we shall briefly survey the analytic results related to the second question raised in §3. We conclude with a look at the open problems in this subject.

2. **Partition ideals.** All of the results in this section are presented (and proved) in PI. We shall, therefore, omit proofs and shall state only those results that are essential to the developments that follow. The lattice-theoretic definitions may be found in [19, Chapter 1].

Let $\mathscr{S}$ denote the set of all sequences $\{f_i\}_{i=1}^{\infty}$ (more briefly $\{f_i\}$), where each f_i is a nonnegative integer and where only finitely many f_i are nonzero. Then $\mathscr{S}$ forms a distributive lattice under the partial ordering $\{f_i\}\leqq\{g_i\}$ whenever $f_i\leqq g_i$ for each i. Furthermore the functions $\#$ and $\sigma:\mathscr{S}\to N$ (where N is the set of nonnegative integers) given by $\#(\{f_i\})=\sum f_i$, $\sigma(\{f_i\})=\sum f_i\cdot i$ are positive valuations on $\mathscr{S}$. We let $\mathbf{0}=\{0, 0, 0, \cdots\}$ denote the constant sequence of zeros.

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