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ON THE EXTENSION OF BOUNDARY INTEGRABLE ALMOST COMPLEX STRUCTURE[1]

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1. **Introduction.** Let $\{M, M'\}$ be a finite Kähler manifold, i.e., M' is a complex Kähler manifold, M is an open submanifold of M' with compact closure, $M_0=bM$, the boundary of M, is a C^∞ submanifold of M', and for each $p \in M_0$ there exists a coordinate neighborhood U of p with real coordinates $t^1, \cdots, t^{2n-1}$, r such that $r(q)<0$ for $q \in U \cap M$ and $r(q)>0$ for $q \in U \cap (M'-M)$. It is assumed that the following conditions hold:

A. For each boundary point the Levi form has at least two positive eigenvalues.

B. There exists a constant $c_0>0$ such that for all $u \in C^{0,q}(\bar{M}, \Theta)$, $q=1,2$ $((2\square-\triangle)u, u) \geqq c_0(u, u)$ where Θ is the holomorphic tangent bundle of M', $C^{p,q}(\bar{M}, \Theta)$ is the space of all C^∞ Θ-valued (p, q)-forms extendible to a neighborhood of $\bar{M}$, $\square$ (resp., $\triangle$) is the complex (resp., the real) Laplacian on $C^{p,q}(\bar{M}, \Theta)$ and $(\ ,\ )$ is the L_2-inner product over M (see [2]).

Then the main result of this note states that a sufficiently small integrable almost-complex structure on M_0 can be extended to a complex structure on M. A complete proof will appear elsewhere; a brief outline follows.

However, we first take a closer look at condition B. Let D be the covariant differentiation operator associated with the connection θ of the metric g on M', i.e.,

$$Du = du + \theta \wedge u = \bar{\partial} u + \tilde{\partial} u$$

for $u \in C^{p,q}(\bar{M}, \Theta)$. Let D^* and $\bar{\partial}^*$ be the formal adjoints of D and $\bar{\partial}$, respectively. Then $\triangle = DD^* + D^*D$ and $\square = \bar{\partial}\bar{\partial}^* + \bar{\partial}^*\bar{\partial}$. Since g is Kähler, $\triangle = 2\square - K$, $K=\sqrt{-1}(e(s)\Lambda - \Lambda e(s))$, where

$$e(s)u = \bar{\partial}\theta \wedge u, \qquad \Lambda u = *^{-1}(\rho \wedge * u),$$

$*$ is the Hodge star operator and ρ is the Kähler form of g. We refer

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