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CYCLIC SUSPENSION OF KNOTS AND PERIODICITY OF SIGNATURE FOR SINGULARITIES[1]

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By a knot we mean a pair (S^m, M^{m-2}) with M^{m-2} a smooth closed oriented submanifold of S^m $(m \geqq 3)$. If such a knot is given and $i: S^m \to S^{m+2}$ is the standard embedding, then one can isotope i in an essentially unique way (Lemma 1 below) to an embedding $j: S^m \to S^{m+2}$ whose intersection with iS^m is $M \subset S^m$ transversally. The n-fold cyclic branched cover of (S^{m+2}, iS^m) branched along (jS^m, M^{m-2}) exists uniquely and is a manifold pair (S_n^{m+2}, M_n^m), where S_n^{m+2} is diffeomorphic to the sphere. This pair we call the *n-fold cyclic suspension* of (S^m, M^{m-2}), or briefly *n-suspension.*

This construction is motivated by the following theorem. Recall that if $g: (C^k, 0) \to (C, 0)$ is a polynomial with isolated singularity at zero, the *link* $K_g \subset S^{2k-1}$ of g is the intersection of $g^{-1}(0)$ with a sufficiently small sphere $S^{2k-1} \subset C^k$ at the origin.

THEOREM 1. *If $g: (C^k, 0) \to (C, 0)$ is a polynomial with isolated singularity at zero, and $f: (C^{k+1}, 0) \to (C, 0)$ is the polynomial $f(z_1, \cdots, z_{k+1}) = g(z_1, \cdots, z_k) + z_{k+1}^n$, then the link (S^{2k+1}, K_f) of f at zero is diffeomorphic to the n-suspension of the link (S^{2k-1}, K_g) of g.*

In particular we get a remarkably simple iterative topological construction of the Brieskorn manifolds [2] as repeated cyclic suspensions of torus links.

The above result has been announced independently by L. Kauffman [4] for weighted homogenious polynomials using an equivalent construction defined for knots whose complement $S^m - M^{m-2}$ fibres over S^1 (*fibred knots*). Another version is due to Bredon [1] when $n=2$.

ADDED IN PROOF. The full construction and Theorem 1 have been found independently by Kauffman (private communication); a more general construction, which for links of isolated singularities of polynomials $f(x)$ and $g(y)$ gives the link for $f(x)+g(y)$, has also been found independently by Kauffman and the author.

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