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SOME EXAMPLES OF SPHERE BUNDLES OVER SPHERES WHICH ARE LOOP SPACES mod p

BY JOHN EWING

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ABSTRACT. In this note we give sufficient conditions that certain sphere bundles over spheres, denoted $B_n(p)$, are of the homotopy type of loop spaces mod p for p an odd prime. The method is to construct a classifying space for the p-profinite completion of $B_n(p)$ by collapsing an Eilenberg-Mac Lane space by the action of a certain finite group.

We say that a space X has some property mod p if the localization of X at p has the property. The problem of determining which spheres are of the homotopy type of loop spaces mod p has been completely solved by Sullivan [9]. It is therefore natural to ask which sphere bundles over spheres are of the homotopy type of loop spaces mod p. In this regard, results of Curtis [2] and Stasheff [7] concerning the question of which sphere bundles over spheres are H-spaces mod p give some negative information. Moreover, in a recent paper [3] we investigated a certain class of sphere bundles over spheres and gave necessary conditions for them to be of the homotopy type of a loop space mod p for p an odd prime. In this note we prove that certain of these bundles satisfying the conditions of [3] are of the homotopy type of a loop space mod p and answer a question posed in [8].

For p an odd prime and n a positive integer, the space $B_n(p)$ is an S^{2n+1}-bundle over $S^{2n+1+2(p-1)}$ classified by the generator of the p-primary part of $\pi_{2n+2(p-1)}(S^{2n+1})$. From [5] we have that $H^*(B_n(p); Z/p)$ is an exterior algebra on generators x and y, where $\deg x=2n+1$, $\deg y=2n+2p-1$ and $\mathscr{P}^1 x=y$. Although few of the $B_n(p)$ are of the homotopy type of a loop space mod p (see [3]), we have the following exceptions.

THEOREM 1. *The space $B_n(p)$ is of the homotopy type of a loop space* mod p *if n and p satisfy any of the following conditions*:

(i) $n=1$; $p=$ *any odd prime*,
(ii) $n=p-2$; $p=$ *any odd prime*,
(iii) $n=7$; $p=17$,
(iv) $n=5$; $p=19$,
(v) $n=19$; $p=41$.

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