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PERTURBATION BY TRACE CLASS OPERATORS[1]

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When are two selfadjoint operators unitarily equivalent modulo the trace class? The version of this theorem in which "trace class" is replaced by "compact" is settled by the Weyl-von Neumann theorem [**1**]: *If A and B are selfadjoint operators, there exists a unitary operator U such that UAU^*-B is compact if and only if A and B have the same essential spectrum.*

The question of trace class equivalence is more delicate and requires a study of additional invariants. Thus the Kato-Rosenblum theorem states: *If A and B are selfadjoint operators for which $A-B$ is trace class, then A and B have unitarily equivalent absolutely continuous parts* [**1**].

Our purpose in the present note is to announce the following answer to the question posed above.

THEOREM. *Two bounded selfadjoint operators A and B are unitarily equivalent modulo the trace class if and only if*

(1) *A_{ac} is unitarily equivalent to B_{ac},*

(2) essential spectrum (A)=essential spectrum (B),

(3) *there exists a decomposition of the sets $\pi_f(A)$ and $\pi_f(B)$, the isolated eigenvalues of A and B of finite multiplicity*

$$\pi_f(A) = \pi_{\approx}(A) \text{ union } \pi_w(A), \qquad \pi_f(B) = \pi_{\approx}(B) \text{ union } \pi_w(B),$$

such that

$$\text{(a)} \qquad \sum_{a_n \text{ in } \pi_{\approx}(A)} d(a_n, \mathrm{sp}_{\mathrm{ess}}(A)) + \sum_{b_n \text{ in } \pi_{\approx}(B)} d(b_n, \mathrm{sp}_{\mathrm{ess}}(B)) < \infty,$$

$$\text{(b)} \qquad \sum_{a_n \text{ in } \pi_w(A)} d(a_n, g(a_n)) < \infty$$

for some one-to-one correspondence g between the points of $\pi_w(A)$ and $\pi_w(B)$.

In this statement it is to be understood that the isolated eigenvalues are counted according to their multiplicity.

The significance of conditions (a) and (b) is that if two bounded selfadjoint operators A and B are unitarily equivalent modulo the trace class,

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