

STUNTED PROJECTIVE SPACES AND THE
 J-ORDER OF THE HOPF BUNDLE

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Let FP^n be the projective space of dimension n over F where $F=C, Q$ (Q stands for quaternions). We have the natural cofibrations

$$FP^{m-1} \rightarrow FP^n \rightarrow FP_m^n$$

for $m \leq n$.

Let H denote the Hopf bundle over FP^n . It is well known that the stunted projective space FP_k^{n+k} can be identified with the Thom space of kH over FP^n , this last being denoted by $(FP^n)^{kH}$.

We study the stable homotopy types of stunted projective spaces FP_k^{n+k} for $F=C$ and $F=Q$ and obtain a complete classification. This classification is given in terms of the J -order of the Hopf bundle over FP^n .

We denote by A_n the J -order of H over CP^n , and by B_n the J -order of H over QP^n . With this notation the results are:

THEOREM A. *The spaces $(CP^n)^{kH}$ and $(CP^n)^{lH}$ are of the same stable homotopy type if and only if one of the following conditions holds: ($n \neq 2, 4$)¹*

- (i) $k-l \equiv O(A_n)$,
- (ii) $k-l \equiv O(A_{n-1})$ and $k+l \equiv O(A_n)$,
- (iii) $k-l \equiv O(A_{n-1})$ and $k+l+2(n+1) \equiv O(A_n)$.

THEOREM B. *The spaces $(QP^n)^{kH}$ and $(QP^n)^{lH}$ are of the same stable homotopy type if and only if one of the following conditions holds:*

- (i) $k-l \equiv O(B_n)$,
- (ii) $k-l \equiv O(B_{n-1})$ and $k+l \equiv O(B_n)$.

In [1] we have proven that the conditions in Theorem A are necessary and observed that $A_n = A_{n-1}$ for n odd which completed the classification

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¹ The cases $n=2, 4$ are also solved but the numerical conditions will not be stated here.