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EXTREMAL LENGTH, REPRODUCING DIFFERENTIALS AND ABEL'S THEOREM

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Let c be a 1-chain on a Riemann surface R and $\Gamma_x(R)$ a closed subspace of $\Gamma_h(R)$, the Hilbert space of square integrable harmonic differential forms on R, then there is a unique $\psi_x(c) \in \Gamma_x(R)$ such that $\int_c \omega = (\omega, \psi_x(c))$ for all $\omega \in \Gamma_x(R)$. $\psi_x(c)$ is called the $\Gamma_x(R)$-reproducing differential for c and $\|\psi_x(c)\|^2$ is a conformal invariant. For the case of a 1-cycle c an extremal length interpretation for the squared norm of the reproducing differential was given by Accola [1] and Blatter [2] for $\Gamma_h(R)$, by Marden [3] for $\Gamma_{ho}(R)$ and by Rodin [5] for $\Gamma_{hse}(R)$. In each of these results the curve family whose extremal length gave the square of the norm of the reproducing differential was a homology class associated with c. Rodin [5] asked whether there were similar theorems for other subspaces of $\Gamma_h(R)$ and what the proper curve family would be in case c was an arbitrary 1-chain, not necessarily a 1-cycle. If c is a single arc, then a reduced extremal distance interpretation of the norm of the reproducing differential for $\Gamma_{he}(R)$, $\Gamma_{hm}(R)$ and $\Gamma_{he}(R) \cap \Gamma^*_{hse}(R)$ was given in [4]. The purpose of this paper is to announce solutions to the problems posed by Rodin for a large number of important subspaces of $\Gamma_h(R)$; a complete, detailed paper is forthcoming.

For the sake of simplicity we shall consider only compact Riemann surfaces; this case gives rise to one of the most important applications. Let c be a 1-chain on the compact Riemann surface R. Suppose that $\partial c = \sum_{j=1}^J n_j b_j - \sum_{i=1}^I m_i a_i$, where the points a_i, b_j are all distinct and m_i, n_j are positive integers, unless $\partial c = 0$. Define $\mathscr{F} = \mathscr{F}(c) = \{d : d$ is a 1-chain on R and $\partial d = \partial c\}$ and $\mathscr{H} = \mathscr{H}(c) = \{d : d \in \mathscr{F}$ and $c - d$ is homologous to $0\}$. Consider fixed local coordinates w_i, z_j defined in a neighborhood of a_i, b_j respectively. Given vectors $\boldsymbol{r} = (r_1, \cdots, r_I)$ and $\boldsymbol{s} = (s_1, \cdots, s_J)$ of positive numbers, let $R(\boldsymbol{r}, \boldsymbol{s})$ be the bordered Riemann surface obtained by removing from R disks of radius r_i, s_j about a_i, b_j,

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