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HOMOLOGY AND IMAGES OF SEMIANALYTIC SETS

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ABSTRACT. The homology of semianalytic sets may be treated using chains which are themselves locally-finite integral combinations of disjoint, oriented semianalytic submanifolds. The analytic image of a relatively compact semianalytic set, though not necessarily semianalytic, admits a finite stratification into connected analytic submanifolds of various dimensions.

A subset A of a (real) analytic manifold M is called *analytic* (respectively, *semianalytic*) if M can be covered by open sets U for which there is a real-valued function f (respectively, a finite family $\mathscr{F}$ of real-valued functions) analytic in U so that $U \cap A$ equals $f^{-1}\{0\}$ (respectively, $U \cap A$ is a union of connected components of $f^{-1}\{0\} \sim g^{-1}\{0\}$ for some $f, g \in \mathscr{F}$). A *stratum* in M is a connected (properly embedded) differentiable submanifold of M. A *stratification* $\mathscr{S}$ of a subset A of M is a locally finite partition of A into strata $\mathscr{S}$ so that $(A \cap \operatorname{Clos} S) \sim S$ is a union of strata in $\mathscr{S}$ having dimension less than the dimension of S. It is well known [**9**, §13], [**7**, 2.8] that every semianalytic set admits a stratification into semianalytic strata.

A j-dimensional *analytic chain* T in M is a sum of integral multiples of oriented j-dimensional semianalytic strata belonging to some fixed stratification of M. Since the restriction to these strata of j-dimensional Hausdorff measure is locally-finite by [**2**, 3.4.8(13)], the analytic chain T is (by oriented integration, counting multiplicities, of differential j forms of compact support in M) a j-dimensional current in M. The set spt T, being the union of the closures of the strata occurring with nonzero multiplicity, is semianalytic. For $j \geqq 1$, the $(j-1)$-dimensional current ∂T, defined by $\partial T(\psi) = T(d\psi)$ for $\psi \in \mathscr{D}^{j-1}(M)$, is, by [**2**, 4.2.28], also an analytic chain in M.

Suppose $M \supset A \supset B$. Using the group of *real analytic cycles* $\mathscr{Z}_j(A, B) = \{T: T$ is a j-dimensional analytic chain of compact support, spt $T \subset A$,

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