BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 80, Number 4, July 1974

ON THE BERGMAN KERNEL AND BIHOLOMORPHIC MAPPINGS OF PSEUDOCONVEX DOMAINS

BY CHARLES FEFFERMAN

Communicated by E. M. Stein, October 30, 1973

THEOREM 1. *Let $D_1, D_2 \subset C^n$ be strictly pseudoconvex domains with smooth boundaries and suppose that $F: D_1 \to D_2$ is biholomorphic (i.e., F is an analytic homeomorphism). Then F extends to a diffeomorphism of the closures, $\bar{F}: \bar{D}_1 \to \bar{D}_2$.*

The main idea in proving Theorem 1 is to study the boundary behavior of geodesics in the Bergman metrics (see [**2**]) of D_1 and D_2. To do so, we use a rather explicit formula for the Bergman kernels of D_1 and D_2. We begin with a few definitions. Let $D=\{z \in C^n | \psi(z)>0\}$ be a strictly pseudoconvex domain, where $\psi \in C^\infty(C^n)$ satisfies grad $\psi \neq 0$ on ∂D.

(1) Let $\mathscr{L}(\omega)$ denote the Levi form, i.e. the quadratic form

$$\mathscr{L}(\omega)\, dz\, \overline{dz} = \sum_{j,k} \frac{\partial^2(-\psi)}{\partial z_j\, \partial \bar{z}_k}\bigg|_\omega dz_j\, \overline{dz}_k$$

restricted to the subspace $\{dz \in C^n | \sum_j (\partial\psi/\partial z_j)|_w\, dz_j = 0\}$ of C^n.

(2) For $\omega_1, \omega_2 \in D$, set $\rho(\omega_1, \omega_2) = |\omega_1 - \omega_2|^2 + |(\omega_2 - \omega_1) \cdot (\partial\psi/\partial\omega)|_{\omega_1}|$. (See [**2**] again.)

(3) A smooth function φ defined on $\bar{D} \times \bar{D}$ has *weight* k (where $k \geqq 0$ is an integer or half-integer) if the following estimate holds.

$$|\varphi(\omega_1, \omega_2)| \leqq C(\psi(\omega_1) + \psi(\omega_2) + \rho(\omega_1, \omega_2))^k$$

(4) Set

$$X(z, \omega) = \psi(\omega) + \sum_j \frac{\partial\psi}{\partial\omega_j}\bigg|_\omega (z_j - \omega_j) + \frac{1}{2} \sum_{j,k} \frac{\partial^2\psi}{\partial\omega_j\, \partial\omega_k}\bigg|_\omega (z_j - \omega_j)(z_k - \omega_k).$$

AMS (MOS) subject classifications (1970). Primary 32F15, 32H10; Secondary 32H15.

Key words and phrases. Pseudoconvex domains, Bergman kernel function,. biholomorphic mapping, Levi form, Bergman metric, boundary behavior of mappings.