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COLIMITS IN TOPOI

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Given a cartesian closed category $\boldsymbol{E}$ with subobject classifier $t:1\to\Omega$, it is shown that the functor $\Omega^{(\ )}:\boldsymbol{E}^{\mathrm{op}}\to\boldsymbol{E}$ is tripleable. Standard results from the theory of triples are then used to show that $\boldsymbol{E}$ has $\boldsymbol{I}$-colimits if and only if it has $\boldsymbol{I}^{\mathrm{op}}$-limits. This gives a new proof of Mikkelsen's theorem which states that $\boldsymbol{E}$ has all finite colimits.

1. **Preliminaries on topoi.** A category $\boldsymbol{E}$ is called an *elementary topos* in **[6]** if $\boldsymbol{E}$ is cartesian closed and has a subobject classifier $t:1\to\Omega$. The reader who is not familiar with these notions is referred to **[3]**, **[4]**, and **[6]** (in **[3]** and **[4]**, the existence of finite limits and colimits is assumed, but we do not make that assumption here). Throughout this paper $\boldsymbol{E}$ will be an elementary topos.

$\boldsymbol{E}$ has finite limits since the existence of binary products and terminal object is assumed in cartesian closedness, and the equalizer of $f, g:A\to B$ can be constructed as the subobject classified by $A\xrightarrow{(f,g)}B\times B\xrightarrow{\delta}\Omega$ where δ is the characteristic morphism of the diagonal $\Delta:B\rightarrowtail B\times B$.

For any object A of $\boldsymbol{E}$ the evaluation morphism $ev_A:\Omega^A\times A\to\Omega$ is the characteristic morphism of a subobject $\varepsilon_A\rightarrowtail\Omega^A\times A$ called the *membership relation.*

If $a:A'\rightarrowtail A$ is a monomorphism in $\boldsymbol{E}$, we get another monomorphism

$$\varepsilon_{A'}\rightarrowtail\Omega^{A'}\times A'\xrightarrow{\Omega^{A'}\times a}\Omega^{A'}\times A$$

whose characteristic morphism $\Omega^{A'}\times A\to\Omega$ corresponds, by exponential adjointness, to a morphism $\Omega^{A'}\to\Omega^A$ which is denoted $\exists_a$ and called the direct image morphism.

The following lemma is fundamental.

LEMMA. *Let*

$$\begin{array}{ccc} B' & \xrightarrow{f'} & A' \\ {\scriptstyle b}\downarrow & & \downarrow{\scriptstyle a} \\ B & \xrightarrow[f]{} & A \end{array}$$

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