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AN INEQUALITY FOR THE DISTRIBUTION OF A SUM OF CERTAIN BANACH SPACE VALUED RANDOM VARIABLES

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1. **Introduction.** Throughout the paper B is a real separable Banach space with norm $\|\cdot\|$, and all measures on B are assumed to be defined on the Borel subsets of B. We denote the topological dual of B by B^*.

A measure μ on B is called a mean zero Gaussian measure if every continuous linear function f on B has a mean zero Gaussian distribution with variance $\int_B [f(x)]^2 \mu(dx)$. The bilinear function T defined on B^* by

$$T(f, g) = \int_B f(x)g(x)\,\mu(dx) \qquad (f, g \in B^*)$$

is called the covariance function of μ. It is well known that a mean zero Gaussian measure on B is uniquely determined by its covariance function.

However, a mean zero Gaussian measure μ on B is also determined by a unique subspace H_μ of B which has a Hilbert space structure. The norm on H_μ will be denoted by $\|\cdot\|_\mu$ and it is known that the B norm $\|\cdot\|$ is weaker than $\|\cdot\|_\mu$ on H_μ. In fact, $\|\cdot\|$ is a measurable norm on H_μ in the sense of [3]. Since $\|\cdot\|$ is weaker than $\|\cdot\|_\mu$ it follows that B^* can be linearly embedded into the dual of H_μ, call it H_μ^*, and identifying H_μ with H_μ^* in the usual way we have $B^* \subseteq H_\mu \subseteq B$. Then by the basic result in [3] the measure μ is the extension of the canonical normal distribution on H_μ to B. We describe this relationship by saying *μ is generated by H_μ*. For details on these matters as well as additional references see [3] and [4].

2. **The basic inequality.** The norm $\|\cdot\|$ on B is *twice directionally differentiable* on $B-\{0\}$ if for $x, y \in B$, $x+ty\neq 0$, we have

$$(d/dt)\,\|x + ty\| = D(x + ty)(y) \tag{2.1}$$

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