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THE EXISTENCE FOR THE SOLUTION OF THE ELLIPTIC CAUCHY PROBLEM

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Let D be a simply connected domain of the $z=x+iy$ plane, whose boundary contains a portion σ of the x-axis. Also let $A(z, \zeta)$, $B(z, \zeta)$ be holomorphic functions for $z, \zeta \in D\cup\sigma\cup\bar{D}$, where $\bar{D}=\{z|\bar{z} \in D\}$. The aim of this note is to announce some recent results on the global existence for the Cauchy problem of the first order linear elliptic equations (in complex normal form):

$$\partial W/\partial \bar{z} = A(z, \bar{z})W + B(z, \bar{z})\bar{W} \tag{1}$$

where

$$\partial/\partial \bar{z} = \tfrac{1}{2}(\partial/\partial x + i\partial/\partial y) \cdot W = u + iv.$$

The notations in this note are taken from Yu [**8**], [**9**]. For general reference to the elliptic Cauchy problem, the reader is referred to the survey article by Payne [**6**], and to equation (1), the reader is referred to Vekua [**7**] and Yu [**8**], [**9**].

By using transformation

$$W(z, \bar{z}) = W_0(z, \bar{z})\exp\int_{\zeta_1}^{\bar{z}} A(z, t)\, dt$$

where ζ_1 is a fixed point in $\bar{D}$, the equation (1) can be reduced to the form

$$\frac{\partial W_0}{\partial \bar{z}} = c(z, \bar{z})\bar{W}_0;\ c(z, \bar{z}) = B(z, \bar{z})\exp\left[\int_{\zeta_1}^{z} A^*(\bar{z}, t)\, dt - \int_{\zeta_1}^{\bar{z}} A(z, t)\, dt\right]. \tag{2}$$

The following integral representation for the solution of (1) in a simply connected domain $G\subset D\cup\sigma\cup\bar{D}$ has been established by Vekua [**7**], and later extended to the boundary ∂G of G by Yu [**9**].

LEMMA 1. *Every solution $W(z)$ of (1) in G, continuous in $G\cup\partial G$, has the integral representation*

$$W(z) = \left\{\phi(z) + \int_{z_0}^{z} \Gamma_1(z, \bar{z}, t, \zeta_0)\phi(t)\, dt + \int_{\zeta_0}^{\bar{z}} \Gamma_2(z, \bar{z}, z_0, \tau)\phi^*(\tau)\, d\tau\right\}\exp\int_{\zeta_1}^{\bar{z}} A(z, t)\, dt, \tag{3}$$

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