QUASICONFORMAL EXTENSION OF HOLOMORPHIC MAPPINGS OF A BALL IN C^n

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Let C^n denote the space of n complex variables $z=(z_1, \cdots, z_n)$ with Euclidean norm $\|z\|$. The open unit ball $\{z \in C^n : \|z\| < 1\}$ is denoted by B^n. We consider holomorphic functions $f(z)=(f_1(z), \cdots, f_n(z))$, $z \in B^n$, from B^n into C^n. The second derivative of such a function is a symmetric bilinear operator, $D^2f(z)(\cdot, \cdot)$ on $C^n \times C^n$, and $D^2f(z)(z, \cdot)$ is the linear operator obtained by restricting $D^2f(z)$ to $z \times C^n$, with matrix representation

$$D^2f(z)(z, \cdot) = \left(\sum_{m=1}^{n} \frac{\partial^2 f_k(z)}{\partial z_j\, \partial z_m} z_m \right), \qquad 1 \leq j, k \leq n.$$

A locally biholomorphic mapping $f(z)$ from a domain $G \subset C^n$ into C^n is said to be K-quasiconformal in G if $\|Df(z)\|^n \leq K|\det Df(z)|$, $z \in G$, where $\| \ \|$ denotes the standard operator norm $\|A\|=\sup\{\|Aw\| : \|w\| \leq 1\}$, $A \in \mathscr{L}(C^n)$.

The purpose of this note is to announce the following n-dimensional $(n \geq 1)$ generalizations of one-dimensional results due to J. Becker [1].

THEOREM. *Let $f(z)$ with $Df(0)=I$ be locally biholomorphic in B^n and satisfy*

$$(1) \qquad (1 - \|z\|^2)\, \|(Df(z))^{-1}D^2f(z)(z, \cdot)\| \leq c, \qquad z \in B^n.$$

If $c \leq 1$ then f is univalent in B^n and

$$\|z\|/(1 + c\, \|z\|)^2 \leq \|f(z)\| \leq \|z\|/(1 - c\, \|z\|)^2, \qquad z \in B^n.$$

If f is K-quasiconformal in B^n and $c < 1$ then f is univalent and continuous in the closed ball, $\bar{B}^n$, and f can be extended to a quasiconformal homeomorphism of R^{2n} onto R^{2n}.

For $n=1$, (1) is $|zf''(z)/f'(z)| \leq c/(1-|z|^2)$, the local biholomorphy implies f is 1-quasiconformal in B^1, and our theorem coincides with

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