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ON DECOMPOSITIONS OF A MULTI-GRAPH INTO SPANNING SUBGRAPHS

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1. Let G be a *multi-graph*, i.e., a finite graph with no loops. $V(G)$ and $E(G)$ denote the *vertex-set* and *edge-set* of G, respectively. For $x \in V(G)$, $d(x, G)$ denotes the *degree* (or *valency*) of x in G and $m(x, G)$ denotes the *multiplicity* of edges at x in G, i.e., the minimum number m such that x is joined to any other vertex in G by at most m edges.

A graph H is called a *spanning subgraph* of G if $V(H)=V(G)$ and $E(H)\subseteq E(G)$. Let k be any positive integer. Let

$$\sigma: G = H_1 \cup H_2 \cup \cdots \cup H_k \tag{1.1}$$

be a *decomposition* of G into k spanning subgraphs so that (1) $H_1, H_2, \cdots, H_k$ are spanning subgraphs of G; (2) $H_1, H_2, \cdots, H_k$ are pairwise edge-disjoint; and, (3) $\bigcup_{1\leqq\alpha\leqq k} E(H_\alpha)=E(G)$. For each $x \in V(G)$, let $\nu(x, \sigma)$ denote the number of subgraphs H_α in σ such that $d(x, H_\alpha)\geqq 1$. Evidently,

$$\nu(x, \sigma) \leqq \min\{k, d(x, G)\} \quad \text{for all } x \in V(G). \tag{1.2}$$

2. Given a multi-graph G and any positive integer k, we consider the problem of determining a decomposition σ of G into k spanning subgraphs such that $\nu(x, \sigma)$ is as large as possible for each vertex $x \in V(G)$. In particular, we have proved the following two theorems.

THEOREM 2.1. *If G is a bipartite graph, then, for every positive integer k, there exists a decomposition σ of G into k spanning subgraphs such that*

$$\nu(x, \sigma) = \min\{k, d(x, G)\} \quad \textit{for all } x \in V(G). \tag{2.1}$$

THEOREM 2.2. *If G is a multi-graph, then, for every positive integer k, there exists a decomposition σ of G into k spanning subgraphs such that*

$$\begin{aligned} \nu(x, \sigma) &\geqq \min\{k - m(x, G), d(x, G)\} \quad \textit{if } d(x, G) \leqq k \\ &\geqq \min\{k, d(x, G) - m(x, G)\} \quad \textit{if } d(x, G) \geqq k, \end{aligned} \tag{2.2}$$

for all $x \in V(G)$.

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