BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 80, Number 3, May 1974

A CHARACTERIZATION OF THE FACTORS OF ORDINARY LINEAR DIFFERENTIAL OPERATORS

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Communicated by Fred Brauer, October 8, 1973

Consider the ordinary differential operator L defined by

$$Ly = y^{(n)} + p_{n-1}y^{(n-1)} + \cdots + p_0 y \quad \text{for } y \in C^n(I) \tag{1}$$

where $p_i \in C^i(I)$ and I is any interval of the real line.

For $1 \leqq k < n$, let D_k denote the class of operators Q of type

$$Qy = y^{(k)} + q_{k-1}y^{(k-1)} + \cdots + q_0 y$$

with $q_i \in C^{n-k}(I)$ for $i=0, \cdots, k-1$.

By $W(y_1, \cdots, y_k)$ we mean the Wronskian of the class C^{k-1} functions $y_1, \cdots, y_k$, i.e. $W(y_1, \cdots, y_k) = \det[y_j^{(i-1)}]$.

In [**4**] it was shown that a necessary and sufficient condition for a factorization $L=RQ$ with $R \in D_{n-k}$, $Q \in D_k$ to hold is:

There exist solutions $y_1, \cdots, y_k$ of $Ly=0$ satisfying

$$W(y_1, \cdots, y_k) \neq 0 \quad \text{on } I. \tag{2}$$

The factor Q has the form:

$$Qy = W(y_1, \cdots, y_k, y)/W(y_1, \cdots, y_k) \quad \text{for all } y \in C^n. \tag{3}$$

Here we announce a characterization of R^*—the formal adjoint of the left factor R.

For a differential operator M denote by $N(M)$ the set of all solutions y of $My=0$.

Assume $y_1, \cdots, y_k$ are in $N(L)$ satisfying (2). Let $y_1, \cdots, y_k, \cdots, y_n$ be a basis of $N(L)$. Define

$$\bar{z}_i = W(y_1, \cdots, \hat{y}_i, \cdots, y_n)/W(y_1, \cdots, y_n) \quad \text{for } i = 1, \cdots, n$$

where the circumflex over y_i indicates that y_i is missing and $\bar{z}$ denotes the conjugate of the complex number z.

THEOREM. *Suppose a factorization $L=RQ$ with Q given by* (3) *holds. Then R is unique and*

$$R^*z = W(z_{k+1}, \cdots, z_n, z)/W(z_{k+1}, \cdots, z_n) \quad \textit{for all } z \in C^n. \tag{4}$$

AMS (MOS) subject classifications (1970). Primary 34A30, 34A05; Secondary 34A01.

Key words and phrases. Ordinary linear differential equations, factoring linear differential operators, Wronskians, Lagrange bilinear form, conjugate solutions, formal adjoint operator.