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APPROXIMATION NUMBERS AND KOLMOGOROFF DIAMETERS OF BOUNDED LINEAR OPERATORS

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0. **Introduction.** In this abstract we state some results concerning the approximation numbers and Kolmogoroff diameters of various operators between Banach spaces. Let $\mathscr{L}(E, F)$ and $\mathscr{F}(E, F)$ denote the bounded linear operators and the closure of the finite rank operators, respectively.

Following Pietsch [8], the nth approximation number $\alpha_n(T)$, of a $T \in \mathscr{L}(E, F)$ is defined as follows:

$$\alpha_n(T) = \inf\{\|T - A\| : \operatorname{rank} A \leqq n\};$$

the nth Kolmogoroff diameter of a $T \in \mathscr{L}(E, F)$ is defined by

$$d_n(T) = \inf\{\|Q_G T\| : \dim G \leqq n\}.$$

Here the infimum is over all subspaces $G \subset F$ and Q_G denotes the canonical quotient map of $F \to F/G$.

It is clear that $\alpha_n(T)$ and $d_n(T)$ are monotone decreasing sequences and that $\lim_n \alpha_n(T) = 0$ if and only if T is the limit of finite rank operators and $\lim_n d_n(T) = 0$ if and only if T is compact.

For a brief discussion of the algebraic and analytic properties of α_n and d_n (and other characteristics of bounded linear operators) see [7]. For compact operators on Hilbert space $\alpha_n(T) = d_n(T)$ and this characteristic has been extensively studied in the book of Gohberg and Kreĭn [1]. For arbitrary Banach spaces the results are few. Some very interesting results can be found in the papers [5] and [6]; related results are also to be found in the classic memoir of Grothendieck, second part [2].

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