BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 80, Number 3, May 1974

DIRECT INTEGRAL THEORY FOR WEIGHTS, AND THE PLANCHEREL FORMULA[1]

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Communicated by S. Feldman, May, 15, 1973

Our object is to outline the principal results obtained in the study of decomposition theory of weights. We begin by studying decomposition theory for left Hilbert algebras, Tomita algebras, and the associated modular automorphism groups. A decomposition theory for K.M.S. weights is developed together with the necessary existence and uniqueness theorems for these decompositions. We next examine K.M.S. weights on separable C^*-algebras. As an application we study the Plancherel problem for separable locally compact groups, generalizing the results of [**6**].

1. **Decomposition theory for left Hilbert algebras.** Throughout, $\mathfrak{A}$ will denote a left Hilbert algebra with fulfillment $\mathfrak{A}''$, Hilbert space completion $\mathscr{H}(\mathfrak{A})$, left and right regular representations π_L and π_R, left and right von Neumann algebras $\mathscr{R}_L(\mathfrak{A})$ and $\mathscr{R}_R(\mathfrak{A})$, sharp operation $\#$, and corresponding modular operator Δ. (See [**4**], [**5**].) Also Γ will denote a locally compact space; by a measure μ on Γ we mean a Radon measure.

DEFINITION 1.1. Given (Γ, μ), let $\mathfrak{A}(\gamma)$ $(\gamma \in \Gamma)$ be a left Hilbert algebra. We say $\gamma \mapsto \mathfrak{A}(\gamma)$ is μ-measurable if there are countably many vector fields $\gamma \mapsto \xi_j(\gamma)$, $j=1, 2, \cdots$ such that:

(i) The fields $\gamma \mapsto \xi_j(\gamma)$ are fundamental for (see [**4**]) the field $\gamma \mapsto \mathscr{H}(\gamma) = \mathscr{H}(\mathfrak{A}(\gamma))$ of Hilbert spaces.

(ii) For $j, k=1, 2, \cdots$ the fields $\gamma \mapsto \xi_j^\#(\gamma)$ and $\gamma \mapsto \xi_j(\gamma)\xi_k(\gamma)$ are measurable.

(iii) For μ-almost all γ, $\{\xi_j(\gamma): j=1, 2, \cdots\}$ are dense in $\mathfrak{A}(\gamma)$ with respect to the $\#$-norm.

Measurability of a field of right Hilbert algebras is defined analogously. Although not necessary for some of our theorems, we restrict to the case where Γ is second countable.

AMS (MOS) subject classifications (1970). Primary 46K15; Secondary 46L10, 46L05, 43A10.

[1] The results in this note will form a portion of the authors doctoral dissertation to be submitted at U.C.L.A. in June 1973. The author extends his thanks to his advisor, Professor M. Takesaki, for his extreme patience and helpfulness, and to Professor R. Blattner for discussions on induced representations.