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A GENERAL APPROACH TO NEWTON'S METHOD FOR BANACH SPACE PROBLEMS WITH EQUALITY CONSTRAINTS

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1. **Introduction.** In this announcement we abstract and preview a theory recently developed by the author for applying Newton's method to constrained problems in infinite-dimensional normed linear spaces. The proofs of these results and a detailed account of the theory will appear elsewhere (see **[10]** and **[11]**).

Consider an operator $Q:X_1 \to X_2$ where X_1 and X_2 are normed linear spaces and functionals $\delta_1, \cdots, \delta_m$ defined on X_1. We are concerned with the constrained problem

$$(1.1) \qquad Q(x) = 0; \quad \text{subject to } \delta_i(x) = 0, \qquad i = 1, \cdots, m.$$

More generally we consider the problem

$$(1.2) \qquad Q(x) \in M; \quad \text{subject to } \delta_i(x) = 0, \qquad i = 1, \cdots, m,$$

where M is a finite-dimensional subspace of X_2.

DEFINITION 1.1. By a normalization for problem (1.2) we mean any operator P defined from X_1 into a normed linear space such that $P(x) = 0 \Leftrightarrow x$ is a solution of problem (1.2).

The desirability of a normalization for problem (1.2) is obvious; namely it allows us to replace a somewhat unorthodox constrained problem with a standard-type unconstrained problem. The purpose of this paper is to present a theory for constructing a normalization for problem (1.2) which readily lends itself to Newton's method. In §3 we show that the following well-known methods for ordinary differential equations are actually special cases of our theory—the integral equation formulation of the initial value problem and the boundary value problem and the method of quasilinearization and superposition. An interesting feature of our general approach is that we do not consider Newton's method applied to a single equation, but construct a different equation at each iteration (i.e., a different normalization).

An obvious way to obtain a normalization for problem (1.1) is to consider the operator P from X_1 into $X_2 \times R^m$ defined by $P(x) = (Q(x), \delta_1(x), \cdots, \delta_m(x))$. Indeed, this is the usual approach when X_1 and

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