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ON EXTENDED STRUCTURES OF A CLOSED OPERATOR RELATED TO SEMIGROUP THEORY AND THE ABSTRACT CAUCHY PROBLEM

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1. After the fundamental treatise on the semigroup theory by Hille and Phillips, several extensions of the semigroup theory and the abstract Cauchy problem have been developed. Three of the basic developments which are related to the results in this announcement are the distributional semigroups (V. Barbu [1], J. Chazarain [4], I. Ciorănescu [5], G. Da Prato and U. Mosco [6], H. O. Fattorini [9], C. Foiaş [10], D. Fujiwara [11], E. Larsson [16], J. L. Lions [17], J. Peetre [19], L. Schwartz [21], Ushijima [23], K. Yoshinaga [24]), the linear differential equations in Banach space (S. G. Kreĭn [15] contains a bibliography up to 1966, recently by R. Beals [2], [3], G. Da Prato and Giusti [7], M. Sova [22]), and the semigroup theory on a locally convex space (H. Komatsu [13], Kômura [14], I. Miyadera [18], K. Yosida [25]).

In this communication, we restrict ourselves to consider only a single closed operator A on a Banach space H in order to simplify the statements of our theorems. The problem is the abstract Cauchy problem (ACP), i.e., to find a solution in H for the differential equation $y'(t)=Ay(t)$ on the interval $[0, \infty)$ with $y(0)=x$ for some $x \in H$. This problem by various authors (for example, Hille and Phillips [12], Kreĭn [15]) is to find a strongly continuous semigroup $T(t)$ on $[0, \infty)$ of continuous operators on H such that $T(t)x$ is the solution of the problem for certain $x \in H$. In 1960, J. Lions proposed the study of the distributional semigroup which gives a distributional solution for the problem $y'(t)-Ay(t)=f(t)$. This extends the solvability of the ACP to a larger class of closed operators. What we are trying to do here is to give a formulation in between the two mentioned above. Formally, the idea is similar to that from the theory of partial differential equations, in the sense that the differential operator is solvable in an extended space which is the completion of the underlying space of the differential operator under a weaker topology.

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