BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 80, Number 2, March 1974

THE SPECTRAL MAPPING THEOREM FOR JOINT APPROXIMATE POINT SPECTRUM

BY M. D. CHOI AND CHANDLER DAVIS

Communicated by Jacob Feldman, August 30, 1973

ABSTRACT. The spectral mapping theorem for joint approximate point spectrum is proved when A is an n-tuple of commuting operators on a Banach space and f is any m-tuple of rational functions for which $f(A)$ is defined.

The purpose of the paper is to show how properties of parts of the joint spectrum can be obtained by use of spaces of sequences of vectors. The method (first used for general Banach spaces in [**4**]) provides very simple proofs of some known results; but the main result is believed to be new.

Throughout the paper, we deal with n-tuples of bounded operators on a Banach space $\mathscr{X}$, whose vectors are denoted by $x, y, \cdots$. It will be convenient to use a symbol such as A to denote an n-tuple of operators and a symbol such as λ to denote an n-tuple of complex numbers: $A=(A_1, \cdots, A_n)$, $\lambda=(\lambda_1, \cdots, \lambda_n)$, $0=(0, \cdots, 0)$. The one-sided spectra of interest in this paper are not defined in terms of existence of Banach-algebra inverses, the context of the papers of Robin Harte ([**5**], [**6**]); they are the somewhat different spectra of the following definition. (However, in the special case that $\mathscr{X}$ is a Hilbert space, our result is a corollary of Harte's.)

DEFINITION. *We say* $0 \in \sigma_p(A)$ *in case there exists a nonzero* $x \in \mathscr{X}$ *such that* $Ax=0$ (*i.e.*, $A_jx=0$ *for* $j=1, \cdots, n$). *We say* $0 \in \sigma_\pi(A)$ *in case for every* $\varepsilon>0$ *there exists a unit vector* $x \in \mathscr{X}$ *such that* $\|Ax\|<\varepsilon$ (*i.e.*, $\|A_jx\|<\varepsilon$ *for* $j=1, \cdots, n$). *We say* $0 \in \sigma_\delta(A_1, \cdots, A_n)$ *in case* $0 \in \sigma_\pi(A_1^*, \cdots, A_n^*)$, *where* A_j^* *denotes the Banach-space adjoint of* A_j. *We say* $\lambda \in \sigma_p(A)$ *in case* $0 \in \sigma_p(A-\lambda)$, *and similarly for* σ_π, σ_δ. *The set* $\sigma_p(A)$ *is called the 'point spectrum' of the n-tuple* A; *the set* $\sigma_\pi(A)$, *its 'approximate point spectrum' or 'left approximate spectrum'; the set* $\sigma_\delta(A)$, *its 'approximate defect spectrum' or 'right approximate spectrum'.*

AMS (MOS) subject classifications (1970). Primary 47A10; Secondary 47A20, 47A60.

Key words and phrases. Spectral mapping theorem, joint spectrum, Berberian extension.