BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 80, Number 2, March 1974

A NOTE ON ANOSOV DIFFEOMORPHISMS

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Communicated by Walter Gottschalk, August 28, 1973

1. **Introduction.** In this note we shall study a class Γ of diffeomorphisms on a compact n-dimensional manifold M. The class Γ will include all diffeomorphisms F with the property that the periodic points of F are dense in M. Our main theorem will give a characterization of those diffeomorphisms in Γ that are Anosov diffeomorphisms.

2. **Statement of results.** Let $F:M\to M$ be a diffeomorphism on a compact n-dimensional manifold M and let $DF:TM\to TM$ be the induced derivative mapping on the tangent bundle of M. The mapping F is said to be an *Anosov diffeomorphism* if the tangent bundle can be decomposed into a continuous Whitney sum $TM=E^s+E^u$, such that

(i) E^s and E^u are invariant under DF;

(ii) $DF:E^s\to E^s$ is contracting, i.e., there exist positive constants K and λ, $\lambda<1$, such that

$$\|DF^m(v)\| \leqq K\lambda^m\|v\| \tag{1}$$

for all $v\in E^s$ and $m\in Z^+$;

(iii) $DF:E^u\to E^u$ is expanding, i.e., there exist positive constants k and μ, $\mu>1$, such that

$$\|DF^m(v)\| \geqq k\mu^m\|v\| \tag{2}$$

for all $v\in E^u$ and $m\in Z^+$, cf. **[1]**, **[3]**, and **[6]**.

Since DF is a homeomorphism, the composed mapping DF^m is defined for all $m\in Z$, and this defines a discrete flow on TM. Similarly F^m is a discrete flow on M, and these flows commute with the natural projection $p:TM\to M$. Now let Γ denote the collection of all diffeomorphisms $F:M\to M$ such that the union of the minimal sets of the flow F^m is dense in M. For example, if the periodic points of F are dense in M, then $F\in\Gamma$.

For any diffeomorphism $F:M\to M$ we define the sets $\mathscr{B}$, $\mathscr{S}$, $\mathscr{U}$ in

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AMS (MOS) subject classifications (1970). Primary 58F15; Secondary 34C35.

[1] The first author was partially supported by U.S. Army Grant DA-ARO-D-31-124-71-G176.

[2] The second author was partially supported by NSF Grant GP-38955.