

PROPERTIES OF THREE ALGEBRAS
RELATED TO L_p -MULTIPLIERS¹

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1. **Introduction.** In this paper we shall announce several properties of certain algebras which arise in the study of L_p -multipliers; detailed proofs will be given elsewhere. Let G be a locally compact abelian group and let Γ denote its dual group. Let $L_p(\Gamma)$ denote the space of p -integrable functions on Γ with respect to Haar measure, and let q denote the index which is conjugate to p . Let

$$A_p(\Gamma) = [L_p(\Gamma) \hat{\otimes} L_q(\Gamma)]/K$$

where K is the kernel of the convolution operator $c: L_p \hat{\otimes} L_q(\Gamma) \rightarrow C_0(\Gamma)$ by $c(f \otimes g)(\gamma) = (f * g)(\gamma)$. $A_p(\Gamma)$ is the p -Fourier algebra which was introduced by Figa-Talamanca in [6] where it was shown that $A_p(\Gamma)^*$ is isometrically isomorphic to $M_p(\Gamma)$, the bounded, translation invariant, linear operators on $L_p(\Gamma)$. Herz [11] showed that $A_p(\Gamma)$ is a Banach algebra under pointwise multiplication; it is known that $A_2(\Gamma) = A(\Gamma) = L_1(G)^\wedge$ and that the inclusions $A_2(\Gamma) \subset A_p(\Gamma) \subset A_1(\Gamma) = C_0(\Gamma)$ are norm decreasing if $1 < p < 2$; see [5], [6], [11] for the basic properties of $A_p(\Gamma)$. Let $B_p(\Gamma)$ denote the algebra of continuous functions f on Γ such that $f(\gamma)h(\gamma) \in A_p(\Gamma)$ whenever $h \in A_p(\Gamma)$. The multiplier algebra $B_p(\Gamma)$ is a Banach algebra in the operator norm. We have studied $B_p(\Gamma)$ in [8], [9]. Fix p in $1 < p < 2$.

Regard $L_1(\Gamma)$ as an algebra of convolution operators on $L_p(\Gamma)$ and let $m_p(\Gamma)$ denote the closure of $L_1(\Gamma)$ in $M_p(\Gamma)$. The first result of this paper says that $B_p(\Gamma)$ is isometrically isomorphic to the dual space $m_p(\Gamma)^*$. In the second result, we use certain properties of $B_p(\Gamma)$ to give a theorem of Eberlein type for $M_p(\Gamma)$. In the final section of the paper, we use

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