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PARAMETRICES AND ESTIMATES FOR THE $\bar{\partial}_b$ COMPLEX ON STRONGLY PSEUDOCONVEX BOUNDARIES

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0. **Introduction.** Here we briefly sketch the background of the problem to be considered, and refer to Folland-Kohn [4] for definitions and proofs.

Let X be the boundary of a strongly pseudoconvex region in a complex manifold of complex dimension $n+1$, or more generally a real manifold of dimension $2n+1$ with a strongly pseudoconvex partially complex structure. We then have the tangential Cauchy-Riemann complex

$$0 \longrightarrow \Lambda^{0,0} \xrightarrow{\bar{\partial}_b} \Lambda^{0,1} \xrightarrow{\bar{\partial}_b} \cdots \xrightarrow{\bar{\partial}_b} \Lambda^{0,n} \longrightarrow 0$$

where $\Lambda^{0,j}$ is the space of j-forms of purely antiholomorphic type. If we impose a Riemannian metric on X, we can form the formal adjoint ϑ_b of $\bar{\partial}_b$ and thence the Laplacian $\Box_b=\bar{\partial}_b\vartheta_b+\vartheta_b\bar{\partial}_b$. $\Box_b$ is nonelliptic; however, according to a theorem of Kohn, for $1\leqq j\leqq n-1$, $\Box_b$ satisfies the estimates

$$(1) \qquad \|\phi\|_{s+1} \leqq c_s(\|\Box_b\phi\|_s + \|\phi\|_0), \qquad s = 0, 1, 2, \cdots,$$

for all $\phi \in \Lambda^{0,j}$ with compact support. (Here $\| \ \|_s$ is the L^2 Sobolev norm of order s.) These estimates imply that $\Box_b$ is hypoelliptic; moreover, if X is compact, the nullspace $\mathcal{N}$ of $\Box_b$ is finite-dimensional and there is an operator G on $\Lambda^{0,j}$ satisfying

$$\|G\phi\|_{s+1} \leqq c_s \|\phi\|_s \qquad (\phi \in \Lambda^{0,j}, s = 0, 1, 2, \cdots)$$

and

$$G\Box_b = \Box_b G = I - P$$

where P is the orthogonal projection onto $\mathcal{N}$.

Kohn's method unfortunately gives no clue as to how to compute G. Our purpose here is to construct G (modulo smoothing operators) as an

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