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ON LOBATCHEWSKY MANIFOLDS

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Let M be a complete, simply connected, n-dimensional Riemannian manifold with sectional curvature $K \leqq 0$. Eberlein in [7] and [9] has given the cone topology and a nice compactification $\bar{M}=M \cup M(\infty)$ of M. The boundary $M(\infty)$ of M is the set of asymptotic classes of geodesics in M. $\bar{M}$ is homeomorphic to the closed unit ball in $\boldsymbol{R}^n$ and $M(\infty)$ is homeomorphic to S^{n-1}. Each isometry ϕ of M extends to a homeomorphism of $\bar{M}$. Elements of the isometry group $I(M)$ can be classified according to their fixed points in $\bar{M}$. ϕ is called elliptic if ϕ has a fixed point in M. ϕ is called parabolic or axial if ϕ has exactly one fixed point or two fixed points in $M(\infty)$ respectively. If any two distinct points in the boundary $M(\infty)$ can be joined by a unique geodesic in M (Axioms I and II), then M is called a Lobatchewsky manifold for convenience. A complete, simply connected Riemannian manifold with sectional curvature $K \leqq c < 0$ is a Lobatchewsky manifold.

In the sequel, we shall consider only Lobatchewsky manifolds M and we shall assume that $I(M)$ acts effectively on M.

The main theorem is a description of complete homogeneous Riemannian manifolds with sectional curvature $K \leqq c < 0$.

THEOREM 1. *Let M be a complete homogeneous Riemannian manifold with sectional curvature $K \leqq c < 0$. Either $I(M)$ has a common fixed point in $M(\infty)$ or M is a noncompact symmetric space of rank one.*

The tool of this paper is the concept of the limit set of a subgroup G of $I(M)$. The limit set $L(G)$ is the intersection with $M(\infty)$ of the closure of any orbit of G in M. The limit set is independent of the choice of the orbit. If A is a closed subset of $M(\infty)$ which contains more than one point and A is invariant under a subgroup G of $I(M)$, then $A \supset L(G)$. The totally geodesic hull $\langle A \rangle$ of a subset A in $M(\infty)$ is the intersection of all totally geodesic submanifolds in M whose boundaries contain A.

Let G be a subgroup of $I(M)$. One obtains classification of $L(G)$ in the following manner: (1) $L(G)$ is empty, (2) $L(G)$ contains one point,

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