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SOBOLEV INEQUALITIES FOR RIEMANNIAN BUNDLES

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1. **Introduction.** Sobolev inequalities play a major role in the study of differential operators and nonlinear functional analysis. The inequalities are the primary tools in the study of the properties of spaces of functions with Sobolev topologies; for example, the Schauder ring theorem. There are theorems involving continuity and closure of composition in such spaces [**3**, Chapter 2]. The latter theorems involve application of the inequalities to vector fields. It is this case and its generalization which this paper studies, where $\boldsymbol{R}^n$ is replaced by an arbitrary Riemannian manifold satisfying certain geometric conditions.

While the Sobolev inequalities over $\boldsymbol{R}^n$ have been known for some time, the usual proofs use transform methods and are therefore hard to generalize. In 1959 Nirenberg [**4**] presented particularly elegant proofs, due to himself and other authors, which could be generalized. These proofs are the basis for the results of this paper.

Throughout, M denotes a complete Riemannian n-dimensional manifold without boundary. The canonical volume form on M is denoted dV. Let $\pi: E \to M$ be a vector bundle with a specified smooth metric (,). ∇ is a connection on E satisfying $d(V, W)x_m = (\nabla_{x_m} V, W) + (V, \nabla_{x_m} W)$, where V and W are sections of E and $x_m \in T_m M$. ∇^n is the iterated covariant derivative. In most applications E is a tensor bundle over M.

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