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BOUNDARIES OF COMPLEX ANALYTIC VARIETIES

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Let M be a compact, odd-dimensional submanifold of complex euclidean space C^n. It is perfectly natural to ask for conditions on M which guarantee that it forms the boundary of a complex submanifold or, more generally, of a complex subvariety in C^n. Whenever M does bound a subvariety V, then V is an integral current of least mass having boundary M, and, moreover, V is the *unique* integral current of least mass having boundary M (see [3] and [7]). Thus, such conditions on M could be interpreted as properties sufficient for uniqueness and regularity of the solution to Plateau's problem.

In the case that M is one-dimensional, there has been a great deal of work on this question, beginning with the fundamental results of John Wermer in 1958 [**11**] (see [**5**] or [**12**] for a survey and bibliography). The purpose of this note is to announce results for manifolds of dimension greater than one. The theorems in this case differ strikingly from those of Wermer, the difference being essentially related to the appearance in several variables of "Hartog's phenomenon". The extension theorem of Bochner (see [**2**] and [**8**]) occurs as the special case where M is the graph of a function defined on the boundary of an open set in C^{n-1}.

Detailed proofs will appear elsewhere.

Let M be a compact, orientable, $(2k-1)$-manifold differentiably embedded in C^n. In order that M be, even locally, the boundary of a k-dimensional complex submanifold V, it is necessary that, for all $z \in M$,

$$\dim_C(T_z(M) \cap iT_z(M)) = k - 1;$$

since $T_z(M) \cap iT_z(M)$ is the orthogonal complement in $T_z(V)$ of the complex line spanned by the normal vector to the boundary M at z. The submanifold M will be called *maximally complex* if the above condition holds at all points; the condition says that the tangent space to M at z has a complex subspace of the maximal possible dimension. (Note that the

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