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AN IMPLICIT FUNCTION THEOREM FOR SMALL DIVISOR PROBLEMS

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1. **Introduction.** Various nonlinear problems, not accessible to standard existence theorems, led to new techniques which allowed the solution of the isometric embedding problem (J. Nash [1]) and stability problems of Hamiltonian systems connected with small divisors (A. N. Kolmogorov, V. I. Arnold, J. Moser [2]–[6]). Subsequently, the underlying ideas were abstracted as implicit function theorems [7]–[10], which however do not cover most small divisor problems. It is the aim of this paper to formulate and prove a simple implicit function theorem also covering many of these problems. The underlying idea is due to H. Rüssmann [11]. Its basic idea is a modification of Newton's method in the framework of linear spaces and not in that of the group of coordinate transformations as it was used in [2]–[6], [14]. The proof of this theorem is elementary; the real difficulty, however, lies in showing that the assumptions can be met in the relevant applications. We mention as a new application the perturbation theory of invariant tori of dimension $m \leqq n$ of globally Hamiltonian diffeomorphisms defined on a $2n$-dimensional symplectic manifold, in which we were able to verify those assumptions. The proof will be published elsewhere. I am indebted to J. Moser for acquainting me with small divisor problems.

2. **Implicit function theorem.** The following set up is prompted by H. Jacobowitz [9] and L. Nirenberg [10]. We consider three one-parameter families of Banach spaces X_σ, Y_σ, Z_σ in the closed unit interval: for $0 \leqq \sigma' \leqq \sigma \leqq 1$,

$$X_0 \supseteq X_{\sigma'} \supseteq X_\sigma \supseteq X_1 \tag{1}$$

(and analogous for Y_σ and Z_σ) and with norms $|\ |_\sigma$ in X_σ, $\|\ \|_\sigma$ in Y_σ and $|\ |_\sigma$ in Z_σ satisfying

$$|f|_{\sigma'} \leqq |f|_\sigma, \qquad \|u\|_{\sigma'} \leqq \|u\|_\sigma, \qquad |z|_{\sigma'} \leqq |z|_\sigma \tag{2}$$

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