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RATIONAL CRITICAL POINTS OF THE REDUCED NORM OF AN ALGEBRA

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1. **Introduction.** For a polynomial function $f:E\to k$, where E is an n-dimensional vector space over the field k, the determination of $(C_f)_k$, the set of critical points of f lying in E, has been of interest historically (see [**5**], and [**7**, Chapter 8]). More recently, its importance has been noted in the theory of Gauss transforms and zeta functions [**6**]. In particular, for the case $E=A$, a central simple algebra over an A-field k, the function of interest is the reduced norm $\nu:A\to k$; it is with respect to this polynomial that the more "classical" zeta function is defined (see [**8**, p. 203]).

The main result in this note is contained in the next section, where $(C_\nu)_k$ is determined for a (not necessarily central) simple algebra whose dimension over k is not divisible by the characteristic of k. In the last section the critical set is determined for any associative algebra with unity, under certain separability conditions.

For the definition of the reduced representation and norm in the general (nonsimple) case, see [**1**]. For the classical structure theorems of algebras, see also [**3**] and [**4**]. The definition of critical sets is understood as in [**6**]. For a simple algebra A over k, the the following notation has been adopted: K is the center of A, so that $A=M_m(D)$, a full matrix algebra over the K-central division algebra D, the dimension of D as a vector space over K is d^2, and $[K:k]=t$; thus d is the index of A and D over K; $r=md$, the degree of A over K; and $n=r^2t$, the (vector space) dimension of A over k. We also have the reduced norms $\nu:A\to k$, $\nu':A\to K$, $\nu^*:D\to k$, and $\nu'':K\to k$ (which is, of course, the field norm here). For the general k-algebra A, we let N denote its radical, and $A_1,\cdots,A_s$ the simple component summands of A/N, each with the appropriate invariants K_i, D_i, m_i, d_i, r_i, t_i, n_i, and reduced norms $\bar\nu:A/N\to k$ and $\nu_i:A_i\to k$, $i=1,\cdots,s$. Also $(d\nu)_z$ is the differential map of the norm at z. $(d\nu)_z(h)$ is the coefficient of t in the polynomial $\nu(z+th)-\nu(z)$, for $h\in A$, $t\in k$; $(d\nu)_z$ is a k-linear functional.

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